Introduction

In this section, the lessons focus on algebraic reasoning. Students explore exponents and scientific notation through the use of patterns. Students learn to solve equations, first by using manipulatives and then by working with symbols. They use modeling to collect data, and they learn to record data so that they can see the relationship that exists among tables, graphs, and equations.

These lessons form an outline for your ARI classes, but you are expected to add other lessons as needed to address the concepts and practice the skills introduced in the *ARI Curriculum Companion*.

Some of the lessons cross grade levels, as indicated by the SOL numbers shown below. This is one method to help students connect the content from grade to grade and to accelerate.

For the lessons in this section, you will need the materials listed at right.

MATERIALS SUMMARY

- Linking cubes
- Highlighters
- Counters
- Empty film canisters
- Color tiles
- Scientific calculators
- Balance scale
- Pennies
- Computers with Internet access
- Algeblocks™
- Graph paper
- Dry erase boards with markers
- Small paper cups
- Construction paper
- Pipe cleaners

Standards of Learning

The following Standards of Learning are addressed in this section:

5.20 The student will analyze the structure of numerical and geometric patterns (how they change or grow) and express the relationship, using words, tables, graphs, or a mathematical sentence. Concrete materials and calculators will be used.

5.21 The student will
- a) investigate and describe the concept of variable;
- b) use a variable expression to represent a given verbal quantitative expression involving one operation; and
- c) write an open sentence to represent a given mathematical relationship, using a variable.

5.22 The student will create a problem situation based on a given open sentence using a single variable.

6.21 The student will investigate, describe, and extend numerical and geometric patterns, including triangular numbers, patterns formed by powers of 10, and arithmetic sequences.

6.22 The student will investigate and describe concepts of positive exponents, perfect squares, square roots, and, for numbers greater than 10, scientific notation. Calculators will be used to develop exponential patterns.

6.23 The student will
- a) model and solve algebraic equations, using concrete materials;
- b) solve one-step linear equations in one variable, involving whole number coefficients and positive rational solutions; and
- c) use the following algebraic terms appropriately: *variable, coefficient, term*, and *equation*.

7.19 The student will represent, analyze, and generalize a variety of patterns, including arithmetic sequences and geometric sequences, with tables, graphs, rules, and words in order to investigate and describe functional relationships.

7.20 The student will write verbal expressions as algebraic expressions and sentences as equations.

7.21 The student will use the following algebraic terms appropriately: *equation, inequality,* and *expression.*

7.22 The student will
- a) solve one-step linear equations and inequalities in one variable with strategies involving inverse operations and integers, using concrete materials, pictorial representations, and paper and pencil; and
- b) solve practical problems requiring the solution of a one-step linear equation.

8.14 The student will
  a) describe and represent relations and functions, using tables, graphs, and rules; and
  b) relate and compare tables, graphs, and rules as different forms of representation for relationships.

8.15 The student will solve two-step equations and inequalities in one variable, using concrete materials, pictorial representations, and paper and pencil.

8.16 The student will graph a linear equation in two variables, in the coordinate plane, using a table of ordered pairs.

8.17 The student will create and solve problems, using proportions, formulas, and functions.

8.18 The student will use the following algebraic terms appropriately: *domain, range*, *independent variable,* and *dependent variable*.

Table of Contents

Lesson plans pertaining to the following Standards of Learning are found in this section. Click (or CTRL+click) on each to jump to that lesson.

✱ SOL 5.20

Prerequisite SOL
4.21

Lesson Summary
Students explore number patterns in a 1–25 chart and determine and describe rules for the patterns. (45 minutes)

Materials
Large, display 1–25 chart
"1–25 Chart" handouts
Highlighters
"Number Machine" worksheets

Warm-up
Display a large 1–25 chart, and have students identify and describe at least three different patterns in it. Answers will vary and may include: rows and columns of 5; the numbers in any given row increase by 1; the digit in the ones place is the same in every column; multiples of 4 create a diagonal line.

Lesson
1. Give each student a copy of the "1–25 Chart" handout and a highlighter. Have the students follow the instructions to highlight five "plus signs" on the chart and to fill in the table.
2. Lead a class discussion of the students' observations about the data they collected in their tables. Focus on helping students create rules. Additionally, discuss the following questions:
   - What do all the sums have in common? (All of them are multiples of 5.)
   - How can you know in advance whether the sum will be even or odd? (If the middle number is even, the sum will be even; if the middle number is odd, the sum will be odd.)
   - How could you find the middle number if you know only the sum? (Divide the sum by 5.)
   - What is the relationship between the middle number and each adjacent number? (The number on the left is the middle number minus 1. The number on the right is the middle number plus 1. The number on the top is the middle number minus 5. The number on the bottom is the middle number plus 5.)

Reflection
Distribute the "Number Machine" worksheet, and ask students to find the rule.

Name: ________________________

1–25 Chart

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

1. Choose a box on the chart that can be the center of a "plus sign," and use a highlighter to highlight that box and the four boxes adjacent to it to create the plus sign. The plus sign must not hang off the edge of the chart.
2. Use the numbers in your plus sign to fill in the first row of the Data Table below.
3. Repeat this process four times in order to have a total of five plus signs and five sets of data in the table.

1–25 Chart Data Table

	Center no.	No. on Left	No. on Right	No. Above	No. Below	Sum of all five Nos.	Sum — odd or even?
Plus sign 1							
Plus sign 2							
Plus sign 3							
Plus sign 4							
Plus sign 5							

1. Examine the data in the table. How can you tell whether the sum of a plus sign's numbers will be even or odd?

2. What do you notice about all the sums in the table?

3. Describe a rule for finding the middle number of a plus sign if you know only the sum of the five numbers.

Name: ANSWER KEY

1–25 Chart Data Table

Numbers will vary. Some possible answers are as follows:

	Center no.	No. on Left	No. on Right	No. Above	No. Below	Sum of all five Nos.	Sum — odd or even?
Plus sign 1	8	7	9	3	13	40	even
Plus sign 2	17	16	18	12	22	85	odd
Plus sign 3	14	13	15	9	19	70	even
Plus sign 4	12	11	13	7	17	60	even
Plus sign 5	19	18	20	14	24	95	odd

1. Examine the data in the table. How can you tell whether the sum of a plus sign's numbers will be even or odd?

   If the middle number is odd, the sum will be odd. If the middle number is even, the sum will be even.

2. What do you notice about all the sums in the table?

   All the sums have a 0 or 5 in the ones place. They are all multiples of 5.

3. Describe a rule for finding the middle number of a plus sign when you know only the sum of the five numbers.

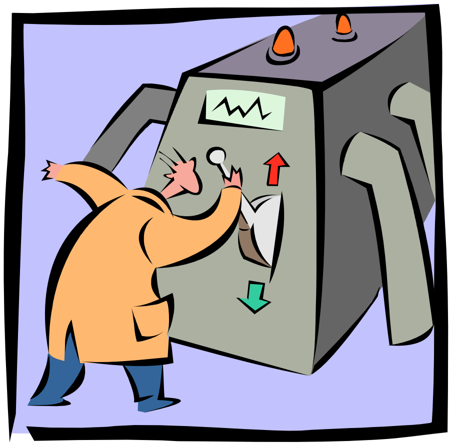
   To find the middle number of a plus sign when you know only the sum of the five numbers, divide the sum by 5, since the sum of the plus sign numbers is five times the middle number.

Name: ________________________

Number Machine

A number machine uses a rule to change input numbers into output numbers. The table below shows what happened when different numbers went into a number machine.

INPUT	OUTPUT
12 →	→ 6
4 →	→ 2
10 →	→ 5
22 →	→ 11



1. Write the rule that you think this number machine used to change the input numbers.

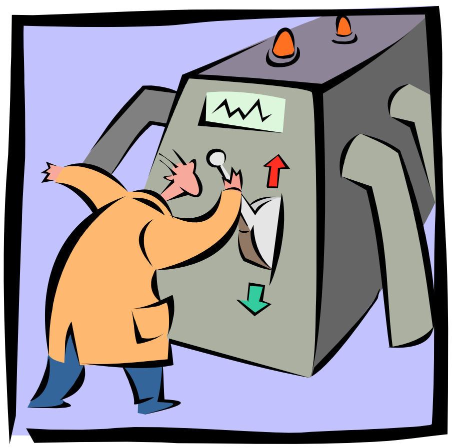
2. Using the same rule, give three more examples of other input numbers and the output numbers that you would get from this machine.

Name: <u>ANSWER KEY</u>

Number Machine

A number machine uses a rule to change input numbers into output numbers. The table below shows what happened when different numbers went into a number machine.

INPUT			OUTPUT
12	→	→	6
4	→	→	2
10	→	→	5
22	→	→	11



1. Write the rule that you think this number machine used to change the input numbers.

   <u>The output number is half of the input number.</u>

2. Using the same rule, give three more examples of other input numbers and the output numbers that you would get from this machine.

   <u>Other examples could include the following:</u>

Input		Output
8	→	4
100	→	50
90	→	45

✱ SOL 5.20

Prerequisite SOL
4.21

Lesson Summary
Students explore number patterns in a hundreds chart and determine and describe rules for the patterns. (45 minutes)

Materials
Large, display hundreds chart
"Hundreds Chart" handouts
"Hundreds Chart Recording Sheet" handouts
"Reflection" worksheets

Vocabulary
variable. A symbol, usually a letter, representing an unknown quantity.
expression. A combination of numbers and/or variables using mathematical operations but no equal sign.

Warm-up
Display a large hundreds chart, and have students identify and describe at least three different patterns in it. Answers will vary and may include: rows and columns of 10; the numbers in any given row increase by 1; the digit in the ones place is the same in every column; multiples of 9 create a diagonal line.

Lesson
1. Give each student a copy of the "Hundreds Chart" handout and the "Hundreds Chart Recording Sheet" handout. Have the students choose any number on the chart and write that number in the first space in the "Arrow" column on the recording sheet. Caution students not to choose a number on the perimeter of the chart.
2. Tell students that they are going to move one space on the chart in a direction indicated by an arrow. The first arrow points to the right, so have the student move one space to the right and write the new number on the recording sheet.
3. Ask students what the relationship is between the number chosen and the number that is one space to the right. (The number to the right is one greater than the original number.) Ask whether this is true for any number on the chart. (Yes) If the students are unsure, have them try some other numbers on the chart.
4. Tell students that they are going to write a rule for determining the number that is one space to the right of any number on the hundreds chart. In order to write the rule so that it can be used for any number on the chart, a variable will be used. Explain that a variable is a letter (in this case, n) that can represent any number chosen.
5. Ask students what needs to be done to the number chosen to find the number that is one space to the right. (Add one to it.) If we use the letter n as our variable to represent the number chosen, what expression could we write to show that we need to add one to that number? ($n + 1$)
6. Have the student write the expression in the "Rule" column on the recording sheet.
7. Repeat steps 1–6 for the seven other arrows.
8. When the recording sheet is complete, have the students practice determining the final number when given a start number and a series of arrows. For the first few examples, allow the student to use the hundreds chart. Then, have the students try it without the chart, but allow them to use the rule sheet.

Reflection
Have students complete the "Reflection" worksheet.

Name: ________________________

Hundreds Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Name: ____________________________

Hundreds Chart Recording Sheet

Arrow	Rule
___ → = ___	_______________ →
___ ← = ___	_______________ ←
___ ↓ = ___	_______________ ↓
___ ↑ = ___	_______________ ↑
___ ↘ = ___	_______________ ↘
___ ↖ = ___	_______________ ↖
___ ↙ = ___	_______________ ↙
___ ↗ = ___	_______________ ↗

Name: ANSWER KEY

Hundreds Chart Recording Sheet

Numbers will vary.

Arrow	Rule
6 → = 7	*n* + 1 →
16 ← = 15	*n* – 1 ←
41 ↓ = 51	*n* + 10 ↓
78 ↑ = 68	*n* – 10 ↑
83 ↘ = 94	*n* + 11 ↘
66 ↖ = 55	*n* – 11 ↖
27 ↙ = 36	*n* + 9 ↙
32 ↗ = 23	*n* – 9 ↗

Name: ________________________

Arrow Math Practice

55 → ↑ →

67 ← ← ↓ ↘

16 ↙ ↙ ←

39 ↖ → ↖ → ↓

82 ↗ ↗ ↗ ← ↘

Name: <u>ANSWER KEY</u>

Arrow Math Practice

55 → ↑ →	<u>47</u>
67 ← ← ↓ ↘	<u>86</u>
16 ↙ ↙ ←	<u>33</u>
39 ↖ → ↖ → ↓	<u>29</u>
82 ↗ ↗ ↗ ← ↘	<u>63</u>

Name: ______________________

Reflection

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Based on the chart above, use a variable to write a rule for the number that is one space away from any number on the chart when you move in the direction indicated by the arrow.

↓ ____________________

↗ ____________________

What does the variable represent in your rule?

Name: <u>ANSWER KEY</u>

Reflection

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Based on the chart above, use a variable to write a rule for the number that is one space away from any number on the chart when you move in the direction indicated by the arrow.

↓ <u>$n + 5$</u>

↗ <u>$n - 4$</u>

What does the variable represent in your rule?

<u>The variable represents the starting number.</u>

✱ SOL 5.21

Prerequisite SOL

5.20

Lesson Summary

Students determine a rule to describe the relationship between the input and output numbers in a table. (30 minutes)

Materials

"Guess My Rule Recording Sheet" handouts

Vocabulary

variable. A symbol, usually a letter, representing an unknown quantity.
expression. A combination of numbers and/or variables using mathematical operations but no equal sign.

Warm-up

Present the following patterns, one at a time, to the students, and have them identify the missing number and explain why that number fits the pattern.

5	10	15	?	25	30	(Answer: 20. The numbers are multiples of 5, *or* the numbers are increasing by 5.)
?	11	9	7	5	3	(Answer: 13. The numbers are decreasing by 2.)
80	40	20	10	?		(Answer: 5. The numbers are divided by 2.)

Lesson

1. Draw a display table on the board similar to the one shown on the "Guess My Rule Recording Sheet."
2. Lead the class in playing a game of "Guess My Rule." Ask students to select a number to put in the input column in the table.
3. Using a predetermined, unstated rule (e.g., double the input number), write the appropriate "output" number in the table.
4. Continue soliciting input numbers from the students and writing the appropriate output numbers according to the secret rule until the students think they know what the rule is. (Students may choose very large numbers at first, but they should come to realize that smaller numbers make the pattern easier to identify.)
5. At that point, supply a number for the input column, and let the students determine the appropriate output number. A correct output number will prove that they have figured out the rule. (Students may guess the rule incorrectly if they guess too quickly or if they attend to only one pair of input-output numbers. For example, if the first input number is 5 and the output number is 10, students may guess that the rule is to add 5 to any input number, when the real rule is to double the input number. When that happens, simply say, "That is not *my* rule," and write the correct output number.)
6. Have students verbalize the rule, e.g., "To get the output number, double the input number." Do not allow the rule to be verbalized until most, if not all, of the students are able to give a correct output number for any given input number.
7. Have the students devise a variable expression to represent the rule they have discovered, e.g. $2 \times n$ or $n + n$. Write the variable expression for the rule in the last cell in the table.
8. Repeat the above activity as many times as you deem necessary, using a different rule each time. Another good example is, "Add 6 to the input number." Multi-step rules can be given for a challenge, e.g., "Multiply the input number by 2 and subtract 5 to get the output number."
9. Distribute the "Guess My Rule Recording Sheet" handouts, and have students work in pairs to play the game, taking turns as the player who creates the rule.

Reflection

Have students complete the “Reflection” worksheet.

Name: ______________________________

Guess My Rule Recording Sheet

Input	Output

Name: ANSWER KEY

Guess My Rule Recording Sheet

The numbers will vary.

Input	Output
5	10
9	18
3	6
12	24
4	8
6	12
n	*n + n*

Name: ______________________

Reflection

Jay and Marissa played a game of "Guess My Rule." When they finished the game, some water was spilled on their table, and all of the numbers written in their table washed away. Jay remembered what the rule was, but neither of them could remember the numbers they wrote.

Fill in the input and output columns of the following table with some numbers that might have been used in their game, using the rule expressed in the last cell.

Input	Output
n	$n - 3$

When you play the "Guess My Rule" game and you are the one who must guess the rule that is being used, how do you do it? Explain your strategy for figuring out the rule.

Name: ANSWER KEY

Reflection

Jay and Marissa played a game of "Guess My Rule." When they finished the game, some water was spilled on their table, and all of the numbers written in their table washed away. Jay remembered what the rule was, but neither of them could remember the numbers they wrote.

Fill in the input and output columns of the following table with some numbers that might have been used in their game, using the rule expressed in the last cell.

Numbers will vary.

Input	Output
5	2
7	4
12	10
3	0
n	$n - 3$

When you play the "Guess My Rule" game and you are the one who must guess the rule that is being used, how do you do it? Explain your strategy for figuring out the rule.

Answers will vary. A possible answer is the following: If the number in the output column is larger than the number in the input column, I know that I must use addition or multiplication for the rule. On the other hand, if the output number is smaller than the input number, then I must use subtraction or division.

✱ SOL 5.21, 6.23

Prerequisite SOL
5.20

Lesson Summary
Students practice writing variable expressions that represent the steps of a number trick. (45 minutes)

Materials
Film canisters	“Recording Sheet” handouts
Counters	“Reflection” handouts

Vocabulary
equation. A mathematical sentence stating that two expressions are equal.
variable. A symbol, usually a letter, representing an unknown quantity.

Warm-up
Present the following number trick to the class:
- Think of a number from 1 to 10.
- Multiply your number by 6.
- Add 12 to the result.
- Multiply by ½.
- Subtract 6.
- Divide by 3.
- Write your answer.

The answer should be the original number. Ask the students to explain how this number trick works. (Answers will vary.) Tell the students that they will be examining a similar number trick in this lesson.

Lesson
1. Distribute the “Number Trick Recording Sheet” handouts, and lead the students through it. Have them use the middle column to record the steps. Everyone should have 2 as the final result.
2. Explore the number trick with students by using the film canisters and counters as models and have them record each step algebraically in the last column. Have each student fill the film canister with his/her chosen number of counters. Ask the students what mathematical symbol can be used to represent this “unknown” number or *variable*. (A lower case letter) Have students write the variable *n* (or any lower case letter) in the last column on the chart.
3. <u>Add 4</u>: Ask students how they could model “add 4.” Have students put 4 counters *with,* not in, the canister. Now, some unknown number (*n*) of counters in the canister and 4 more counters are grouped together. Ask how this can be recorded algebraically. ($n + 4$)
4. <u>Multiply by 2</u>: Ask students how they could model “times 2.” (Add another filled canister grouped with 4 more counters) Ask how this can be recorded algebraically. [Several ways: $2(n + 4)$ or $(2 \times n) + (2 \times 4)$ or $2n + 8$.] Discuss the different notations.
5. <u>Subtract 4</u>: Ask students how they could model “subtract 4.” (Take away 4 of the counters.) Ask how this can be recorded algebraically. (Use “ – 4” at the end of the expression written in step 3.)
6. <u>Divide by 2</u>: Ask students how they could model this “divide by 2.” (Make 2 equal groups with the remaining 2 canisters and 4 counters to show the division process.) Ask how this can be recorded. (Use 2 as the denominator and the expression from step 4 as the numerator; then simplify.)
7. <u>Subtract the original number</u>: Ask students how they could model this operation. (Remove the canister.) Ask how could this be recorded. (Subtract the variable.)

Reflection
Have students complete the “Reflection” worksheet.

Name: ________________________

Number Trick Recording Sheet

Verbal	Numerical	Algebraic
Think of a number from 1 to 10.		
Add 4 to your number.		
Multiply the result by 2.		
Subtract 4.		
Divide by 2.		
Subtract the original number.		
Write your answer.		

Name: <u>ANSWER KEY</u>

Number Trick Recording Sheet

Verbal	Numerical	Algebraic
Think of a number from 1 to 10.	9	n
Add 4 to your number.	$9 + 4 = 13$	$n + 4$
Multiply the result by 2.	$13 \times 2 = 26$	$(n + 4) \times 2 = 2n + 8$
Subtract 4.	$26 - 4 = 22$	$2n + 8 - 4 = 2n - 4$
Divide by 2.	$22 \div 2 = 11$	$\frac{2n - 4}{2} = n - 2$
Subtract the original number.	$11 - 9 = 2$	$n - 2 - n$
Write your answer.	2	2

Name: ______________________

Reflection

1. Match each of the following variable expressions:

$s + 2$	Half of some number
$t - 8$	Two more than some number
$\frac{y}{2}$	Four times some number
$4n$	Eight less than some number

2. Define the term *variable*.

3. Explain how a variable is used in mathematics.

Name: <u>ANSWER KEY</u>

Reflection

1. Match each of the following variable expressions:

$s + 2$	Half of some number
$t - 8$	Two more than some number
$\frac{y}{2}$	Four times some number
$4n$	Eight less than some number

2. Define the term *variable*.

   <u>A variable is a symbol, usually a letter, representing an unknown quantity.</u>

3. Explain how a variable is used in mathematics.

   <u>A variable is used to write a rule that is true for any number that replaces the variable.</u>

✱ SOL 5.22

Prerequisite SOL
5.21

Lesson Summary
Students practice matching open sentences with contextual situations. (45 minutes)

Materials
“Recording Sheet” handouts
Small pieces of paper with the word *some* written on them
Matching cards
“Reflection” worksheets

Vocabulary
variable. A symbol, usually a letter, representing an unknown quantity.

Warm-up
Play a class game of “Guess My Rule” (see previous lesson), using the rule “Multiply the input number by 3.” (For example, if the students’ input number is 5, respond with an output number of 15.) Play again, using another rule, e.g., “Double the input number and subtract 1.”

Lesson
1. Present the first number story on the recording sheet to the students. Have the students write an equation to match the story. (15 + 8 = 23) Ask the students how they knew to use addition. (The action in the story implies addition.) Using a small piece of paper with the word “some” written on it, cover the number 15 in the number story. Ask students what they would do if they did not know this number. How could they write an equation for the story? (Use a variable to represent the unknown number.)
2. Have students write equations using variables. ($p + 8 = 23$) Look at the second number story on the recording sheet. Ask students what the unknown number is in this story. (The number of cookies that were eaten.) Ask them to name the mathematical operation implied by the action in this story. (Most likely the students will say subtraction because some of the cookies are eaten or “taken away.”) Ask students to write an equation for this story. (Students should recognize that this equation cannot be completed because there is no way to determine the final result of the action. Explain to the students that they will need to write an open sentence to represent this story: $12 – n =$.)
3. Give the students sets of the matching cards, and have them read the cards to find an open sentence that matches each story problem.

Reflection
Have students complete the “Reflection” worksheet.

Name: ______________________

Recording Sheet

1. James had 15 pennies. He found 8 more. Now James has 23 pennies.

2. Bill bought 12 cookies at the bakery. He ate some of the cookies.

Matching Cards (copy and cut apart)

Tim is 4 years older than Bob.	$b + 4 =$
Mrs. Wilson planted some flowers in her garden. Five of the flowers died.	$n – 5 =$
Sue has 4 fewer books than Brenda.	$b – 4 =$
Tony is collecting baseball cards. So far he has collected 9 cards. His friend gave him some more cards.	$9 + r =$

There are some tables in the cafeteria. Each table can seat 5 students.	$n \times 5 =$
Mom gave Ted some cookies. He asked for 5 more.	$s + 5 =$
Miguel had some boxes of cereal on his shelf. He donated 9 boxes to the homeless.	$r - 9 =$
Mr. Jones has some students in his class. He wants to make 5 teams for field day.	$\frac{n}{5} =$

Name: ________________________

Reflection

1. Write a story that matches the following open sentence:

   $s - 15 =$

2. Write an open sentence that matches the following story:

   Taylor has some vases of flowers. Each vase has 5 flowers.

Name: <u>ANSWER KEY</u>

Reflection

1. Write a story that matches the following open sentence:

   $s - 15 =$

   <u>Answers will vary, for example: "There were some birds on a fence. 15 of the birds flew away."</u>

2. Write an open sentence that matches the following story:

   Taylor has some vases of flowers. Each vase has 5 flowers.

   <u>$a \times 5 =$</u>

✱ SOL 6.21

Prerequisite SOL
5.20

Lesson Summary
Students explore a geometric pattern with color tiles and generate the triangular numbers. (30 minutes)

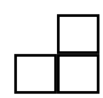
Materials
Color tiles
"Tile Towers Recording Sheet" handouts
"Reflection" worksheets

Warm-up
Give the students a collection of color tiles. Ask them to create a pattern, using the tiles. It is likely that the students will make repeating patterns — i.e., patterns composed of a core that repeats without changing, such as red tile, blue tile, red tile, blue tile.* Ask the students to describe their patterns and explain how to predict what comes next in the pattern. (Once the core that is going to be repeated is established, use that as a reference to determine what comes next after each figure in the pattern.) Explain to the students that this type of pattern is called a "repeating pattern."

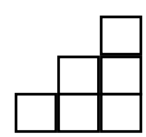
Arrange some tiles to show a growing pattern, e.g., one red tile, a row of 2 red tiles, a row of 3 red tiles, a row of 4 red tiles. Ask students to identify what comes next in this pattern. (A row of 5 red tiles) Ask students to explain how they figured this out. (The number of tiles in each new row is increasing by 1.) Ask the students to determine how this is different from a repeating pattern. (This pattern is changing: the number of tiles is getting larger.) Explain to the students that this type of pattern is called a "growing pattern."

* If students create growing patterns originally, reverse the steps described above.

Lesson
1. Tell the students that they are going to make growing patterns. The first stage of the pattern uses one tile. Have the students place one tile on the table.
2. Give students recording sheets to record each step or stage of the pattern. Have them record the stage number (1), what it looks like (one tile alone), and the number of tiles (1) for that stage of the pattern.
3. Show the students how to build stage 2 of the pattern next to their stage 1. Have them fill in the recording sheet for stage 2.



4. Show the students how to build stage 3 of the pattern. Have them fill in the recording sheet for stage 3.



5. Ask students to predict what the next stage of the pattern will look like. At this point, students should be able to describe the next stage. The descriptions may vary. Students may see the "layers" of tiles growing from the bottom, from the side, or diagonally. If a student is still having trouble identifying the pattern, show the next stage and have the students draw it. The student should be able to predict the number of tiles for the next stage.
6. Once students are able to predict what the next stage will look like and how many tiles it will take, have them build that stage to check their predictions.

Reflection
Have students complete the "Reflection" worksheet.

Name: ________________________

Tile Towers Recording Sheet

Stage #	What I See	# of Tiles
1		
2		
3		
4		
5		
6		

Write a description of the pattern:

Name: ANSWER KEY

Tile Towers Recording Sheet

Stage #	What I See	# of Tiles
1	1	1
2	1 + 2	2
3	1 + 2 + 3	6
4	1 + 2 + 3 + 4	10
5	1 + 2 + 3 + 4 + 5	15
6	1 + 2 + 3 + 4 + 5 + 6	21

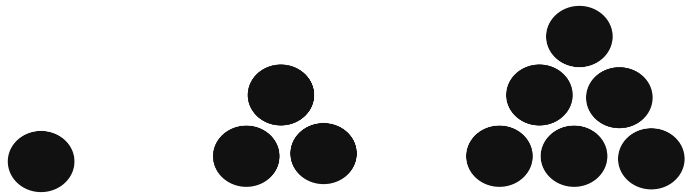
Write a description of the pattern:

Answers will vary, for example: "Each stage of the pattern adds a new column on the right that is one tile taller than the last column of the previous stage."

Name: ______________________

Reflection

Draw a picture to show what the next stage of this pattern will look like:



Stage 1 **Stage 2** **Stage 3** **Stage 4**

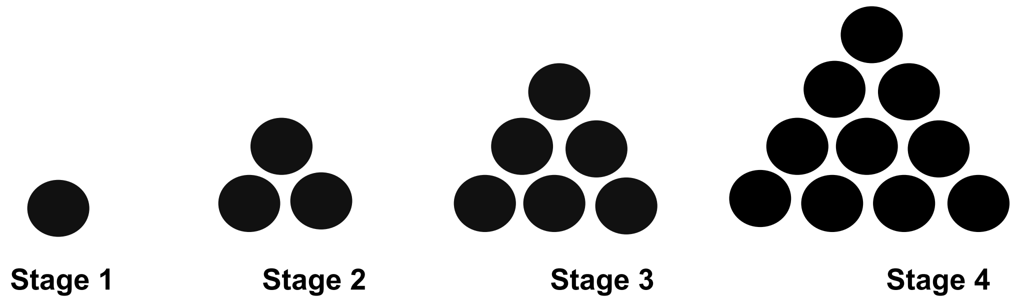
How can you predict the number of circles needed to build the next stage of the pattern?

Write a description of how this pattern is growing.

Name: ANSWER KEY

Reflection

Draw a picture to show what the next stage of this pattern will look like:



How can you predict the number of circles needed to build the next stage of the pattern?
Answers will vary, for example: "To find the number of circles needed for the next stage, add the number that is one more than the number of circles in the bottom row of the previous stage.

Write a description of how this pattern is growing.
(Answers will vary.)

✱ SOL 6.21, 6.22

Prerequisite SOL
5.20

Lesson Summary
Students explore powers of ten and exponents, using a scientific calculator. (30 minutes)

Materials
Scientific calculators
"Calculating Tens Recording Sheet" handouts
"Reflection" worksheets

Vocabulary
base. A factor that is multiplied by itself as many times as indicated by its exponent, i.e., *a* in the expression a^x.
exponent. A number that represents the number of times a base is used as a factor, i.e., *x* in the expression a^x.
factor. In multiplication, each of the numbers that are multiplied to get a product.
product. The answer in multiplication.

Warm-up
Present the following patterns to the students:

5 50 500 5,000 ?
13 130 1,300 13,000 ?

Ask students to identify the next number in each pattern. (50,000 and 130,000) Ask how these patterns are alike. (Possible answers: "Each number has one more 0 than the number before it. Each number has one more place-value position than the number before it. You multiply a number by 10 to find the number that comes next.")

Lesson
1. Distribute calculators and copies of the "Calculating Tens Recording Sheet" handout.
2. Explain the directions for using the calculator to complete the recording sheet, as follows: First, enter the number 10 on the calculator; the result appears in the display window. Note on the recording sheet that to "compute" the product for 10, we simply enter 10 on the calculator. The final column is a way to write 10 in exponential form. Explain that a number written in exponent form has a base number and an exponent. The base number is the factor that is being multiplied. The exponent tells how many times the base number is used as a factor. In this case, the base number is used once; therefore, the exponent is 1.
3. For the second row of the table, have students find the result for the computation 10 x 10 by pressing the "X" key, entering the number 10, and then pressing the "=" key. Have them write the product (from the display window) in the table. (100) Ask students to identify the base number (10) and the exponent (2) and write them in the last column. Ask the students what the 2 represents.
4. To complete the third row (and each succeeding row), have students simply press the "=" key to multiply the previous result by another 10. Have the students complete the recording sheet and discuss the results.

Reflection
Have students complete the "Reflection" worksheet.

Name: ________________________

Calculating Tens Recording Sheet

Compute	Enter on the Calculator	Product	Exponent Form
10 x 1	10 x 1	10	10^1
10 x 10	x 10 =		10^2
10 x 10 x 10	=		10^3
	=		
	=		
	=		
	=		

What patterns do you notice?

Name: ANSWER KEY

Calculating Tens Recording Sheet

Compute	Enter on the Calculator	Product	Exponent Form
10 x 1	10 x 1	10	10^1
10 x 10	x 10 =	100	10^2
10 x 10 x 10	=	1,000	10^3
10 x 10 x 10 x 10	=	10,000	10^4
10 x 10 x 10 x 10 x 10	=	100,000	10^5
10 x 10 x 10 x 10 x 10 x 10	=	1,000,000	10^6
10 x 10 x 10 x 10 x 10 x 10 x 10	=	10,000,000	10^7

What patterns do you notice?

Each time you multiply by an additional 10, the number of zeros in the result increases by 1 and the number of zeros is equal to the exponent.

Name: ______________________

Reflection

1. Write the number 10^9 two other ways.

2. In the number 10^4, identify the base and the exponent, and tell what each represents.

Name: ANSWER KEY

Reflection

1. Write the number 10^9 two other ways.

   1,000,000,000

   10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10

2. In the number 10^4, identify the base and the exponent, and tell what each represents.

   The base is 10 and the exponent is 4. The base is the factor and the exponent is the number of times that the factor is multiplied by itself: 10 x 10 x 10 x 10.

✱ SOL 6.22

Prerequisite SOL
5.8, 5.15, 5.20. 5.21

Lesson Summary
Students build, describe, and explore color-tile patterns that represent square numbers and square roots. (45 minutes)

Materials
Color tiles
“Build a Square Recording Sheet” handouts
“Reflection” worksheets

Vocabulary
square number. A number that results from multiplying any whole number by itself.
square root. A number which, when multiplied by itself, produces a square number.
area. The number of square units needed to cover a two-dimensional figure.

Warm-up
Give students a collection of color tiles, and ask them to build as many different rectangles as possible, using 12 tiles at a time. All rectangles must be made with exactly 12 tiles. Ask: “What are the dimensions of each of the rectangles?” (1 x 12, 2 x 6, 3 x 4) Ask: “What is the area of each of the rectangles?” (12)

Lesson
1. Tell the students that you want them to use the color tiles to build the smallest square figure possible. Because the smallest square is simply one tile, if a student builds a square that is larger than one tile, challenge him/her to think about how to represent a *smaller* square. Ask students to explain how they know whether the figure is a square. (Answers should include all or some of the following: All four sides are congruent. All four corners are right angles. The opposite sides of the figure are parallel. Adjacent sides of the figure are perpendicular.) If a student builds a rectangle, review the properties of a square.
2. Give students recording sheets, and have them record the data from the smallest square. Explain that the length of one side of the tile will be considered 1 unit. The first column will be used to record the number of units on one side of the square figure. (1) The dimensions column will be used to record the number of units along the width times the number of units along the length. (1x1) The last column will be used to record the area of the square — the total number of tiles needed to build the square. (1)
3. Ask students to build the next smallest square possible. (This should be a 2 x 2 square, using four tiles. Again, if a student builds a rectangle, review the properties of a square. If a student builds a larger square, challenge him/her to find one that is smaller. Have students fill in the data for the second square on their recording sheet.
4. Continue this process, having students build successively larger squares and fill in the recording sheet until they have completed a 6 x 6 square.
5. Ask the students what patterns they see in the table. (Answers might include, but are not limited to, the following: The length of one side of each square is increasing by one unit as the squares get larger. The length and width dimensions of each square are always the same. The area of each square is always the product of the length of one side times itself. The area of each successive square is increasing by consecutive odd numbers.)
6. If no one mentions that the numbers in the last column are all square numbers, extend the discussion to square roots, and point out on the chart that the square root of an area measure of a square is equal to the length of one side of the square (column 1). Ask the students to explain why this

relationship exists. Explain that “squaring a number” and “taking the square root of a number” are inverse operations. Have students predict what the next square number (and the square root of that number) on their recording sheet will be.

Reflection

Have students complete the “Reflection” worksheet.

Name: ________________________

Build a Square Recording Sheet

Length of One Side of the Square	Dimensions of the Square	Area of the Square (Total Number of Tiles)

What patterns do you see?

What is your prediction of the numbers for the next row?

Name: ANSWER KEY

Build a Square Recording Sheet

Length of One Side of the Square	Dimensions of the Square	Area of the Square (Total Number of Tiles)
1	1 by 1	1
2	2 by 2	4
3	3 by 3	9
4	4 by 4	16
5	5 by 5	25
6	6 by 6	36

What patterns do you see?

The length of one side of each square is increasing by one unit each time.
The length and width dimensions of each square are always the same.
The area of each square is always the product of the length of one side times itself.
The area of each successive square is increasing by consecutive odd numbers.

What is your prediction of the numbers for the next row?

Length of One Side: 7
Dimensions: 7 by 7
Area: 49

Name: ______________________

Reflection

1. Which of these numbers is a square number: 54, 63, 81, 90?

   ____

   Explain how you know.

   ____

   ____

   ____

2. The number 11 is the square root of which of these numbers: 22, 99, 110, 121?

   ____

   Explain how you know.

   ____

   ____

   ____

Name: ANSWER KEY

Reflection

1. Which of these numbers is a square number: 54, 63, 81, 90?

   __81___

   Explain how you know.

   The number 81 is a perfect square because 9 x 9 = 81. With 81 tiles, you could build a square figure that is made up of 9 rows of tiles with 9 tiles in each row.

2. The number 11 is the square root of which of these numbers: 22, 99, 110, 121?

   __121___

   Explain how you know.

   The number 11 is the square root of 121 because 11 x 11 = 121. Using 121 tiles, you could build a square figure that is made up of 11 rows of tiles with 11 tiles in each row.

✱ SOL 6.23

Prerequisite SOL
5.20, 5.21, 5.22

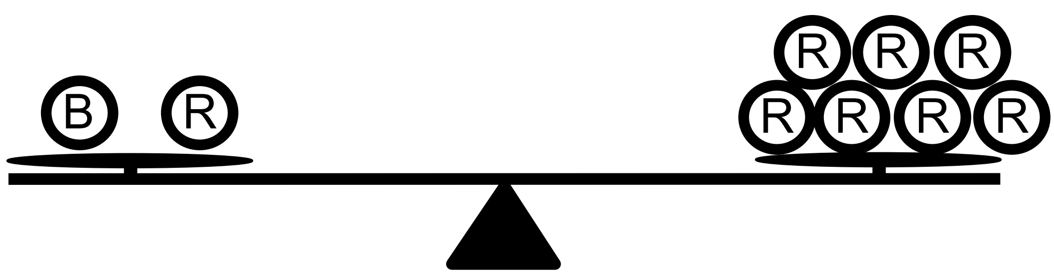
Lesson Summary
Students represent and solve one-step equations, using a balance mat and counters. (45 minutes)

Materials
Balance scales	“Balance Mat” worksheets
20 pennies	Red and blue counters

Vocabulary
equation. A mathematical sentence stating that two expressions are equal.
variable. A symbol, usually a letter, representing an unknown quantity.
term. Parts of an expression separated by addition or subtraction signs.

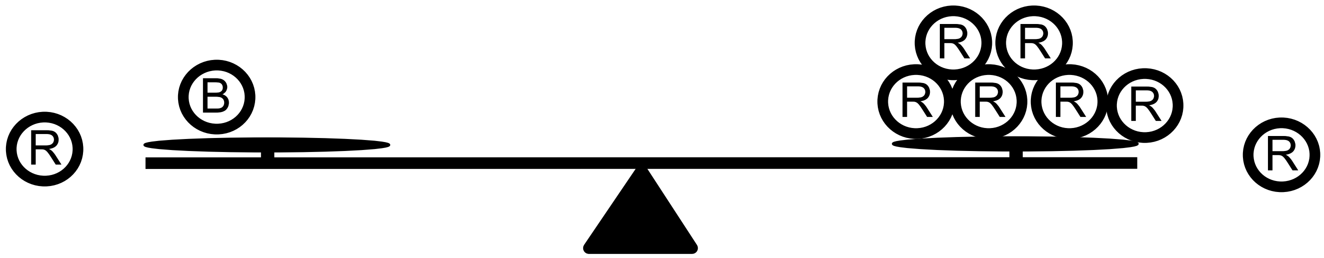
Warm-up
Place 10 pennies on one side of a balance scale and 5 pennies on the other side. Ask if the scale is balanced and why or why not. Students should answer “no” because one side has more pennies and is therefore heavier than the other side. Ask what could be done to make this scale balanced. (Put 5 more pennies on the side that is lighter.) Ask students to explain what they know about the weight of two objects when the scale is balanced. (They have the same weight.)

Lesson
1. Distribute a “Balance Mat” worksheet and counters to each student.
2. Tell students that each red counter has a value of 1 unit. Explain that students are going to use what they know about the red counters and about balance and equality to determine the value of the blue counter.
3. Have students place 1 blue counter and 1 red counter on the left side of the balance shown on their mat and 7 red counters on the right side, as shown:

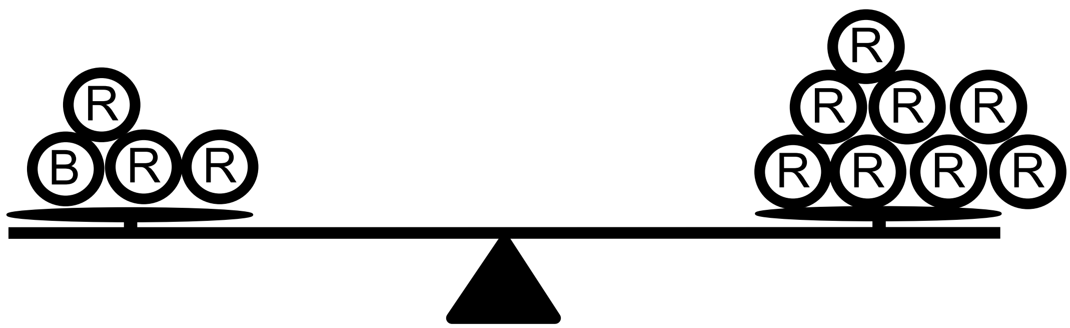


4. Ask students what they can see about the value of the red and blue counters on their mats. (The scale is balanced, so the counters on either side must have an equal value.)
5. Ask for suggestions on how to write an equation to represent what is shown on the balance scale. (The scale is balanced, which indicates that what is on the left side is equal to what is on the right side. Write an equal sign. On the left side, use a variable, *n*, to represent the blue counter since you do not know its value. The red counter on the left side has a value of 1, and this can be recorded as “+ 1” after the *n*. The 7 red counters on the right side have a value of 1 each, and these can be represented by the number 7 on the right side of the equal sign. So the equation is: $n + 1 = 7$.
6. Discuss the meaning of the vocabulary words *equation, variable,* and *term.*
7. Encourage the students to develop a strategy for finding the value of the blue counters. If the students have difficulty, ask them what will happen when you take the one red counter off of the left

side of the balance. (The scale will be out of balance.) Ask what they would need to remove from the right side in order to maintain the balance. (One red counter)



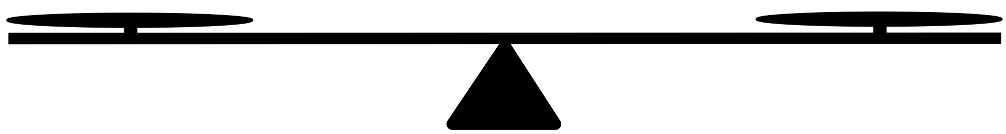
8. Ask students to represent changes in the equation. (Subtract 1 from each side of the equation: $n + 1 - 1 = 7 - 1$.) This leaves one blue counter balanced with 6 red counters. Therefore, one blue is equal to 6 red, or $n = 6$.
9. Have the students use the counters and the balance mat to solve another equation, $x + 3 = 8$, using the process above. Students should set up their balance scale to represent the equation as follows:



Reflection

Have students complete the “Reflection” worksheet.

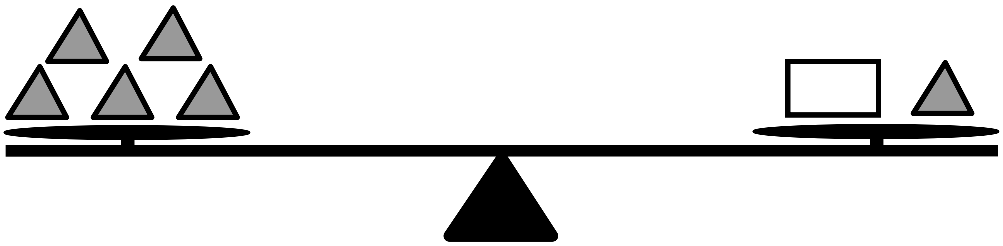
Balance Mat



Name: ______________________

Reflection

If the weight of each gray triangle below is 1 unit, write an equation for the balance scale.



Equation: _______________________

Identify the variables and terms in your equation:

variables: ________________

terms: ________________

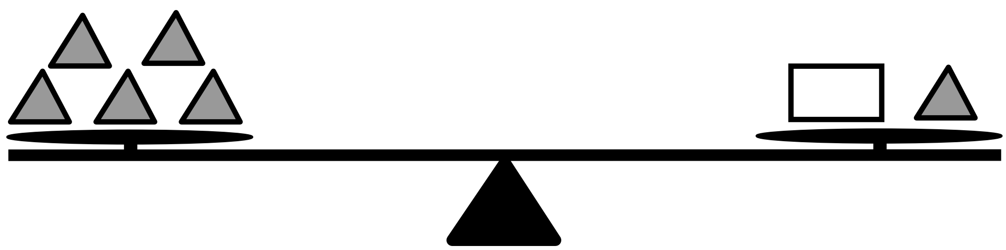
What is the value of the rectangle? _______

Explain how you found your answer:

Name: <u>ANSWER KEY</u>

Reflection

If the weight of each gray triangle below is 1 unit, write an equation for the balance scale.



Equation: <u>5 = *r* + 1</u>

Identify the variables and terms in your equation:

variables: <u>*r*</u>

terms: <u>*r* is a term, 1 is a term, and 5 is a term.</u>

What is the value of the rectangle? <u>4 units</u>

Explain how you found your answer:
<u>If you remove 1 triangle from each side of the scale, you are left with 4 triangles on one side and 1 rectangle on the other side. Therefore, 1 rectangle is equal to 4 triangles (4 units).</u>

✱ SOL 6.22

Prerequisite SOL

6.21

Lesson Summary

Using the data from books, students practice writing numbers in scientific notation. (30 minutes)

Materials

A book with large numbers, such as *On Beyond A Million: An Amazing Math Journey* by David M. Schwartz.

Vocabulary

base. A factor that is multiplied by itself as many times as indicated by its exponent, i.e., *a* in the expression a^x.

exponent. A number that represents the number of times a base is used as a factor, i.e., *x* in the expression a^x.

scientific notation. A system for writing very large or very small numbers as a number between 1 and 10 multiplied by a power of 10.

Warm-up

Have students extract some very large numbers from a book such as *On Beyond a Million: An Amazing Math Journey* by David M. Schwartz, or pull number facts from *The Guinness Book of World Records* or *Earth Facts*. Tell the students to write the number "googol" (1 followed by 100 zeros, or 10^{100}), and time how long it takes them to do this. Ask students to explain the value of using exponents for writing large numbers. (It is a faster way of writing the number.) Review using a base and exponent for writing other large numbers (e.g., million, billion, and trillion) as powers of 10.

Lesson

1. Using examples of large numbers from the selected book, explain that many people work with huge numbers. An example are scientists who measure distances among stars. Very large numbers are so big that they are hard to read and are cumbersome to write. Therefore, when scientists write very big numbers, they take a shortcut and use *scientific notation,* which is different from the usual way of writing numbers (standard notation).
2. Explain the process of changing a huge number into scientific notation. For example, to write in scientific notation the number of Tootsie Rolls® manufactured daily (37,000,000), do the following:
   - <u>Determine the new number between 1 and 10</u>: Move the decimal point in the original number so that the new number is between 1 and 10. (3.7000000, or 3.7)
   - <u>Determine the power of 10</u>: Count the number of places you moved the decimal point. (7) This number equals the exponent. Write the power of 10 that you would need to multiply the new number by in order to get the original number. (10^7)
   - Write the two parts as a multiplication expression. (3.7×10^7)
3. Have students practice writing other numbers from the selected book in scientific notation, for example: the number of people in the U.S. (2.5×10^8); the number of stars in the Milky Way (3×10^{11}); the weight of the earth in pounds (13×10^{25}).
4. Have students practice changing numbers in scientific notation into standard notation.
5. Ask students to find a very large number that interests them, such as the number of M&M's manufactured in one day, and write it in scientific notation.

Refection

Have students complete the questions on the "Reflection" worksheet.

Name: ______________________

Reflection

1. Write the number 243,000,000,000 in scientific notation.

2. Write the number 3.6×10^{12} in standard notation.

3. Explain the advantage of writing numbers in scientific notation.

4. Explain the process for writing numbers in scientific notation.

Name: ANSWER KEY

Reflection

1. Write the number 243,000,000,000 in scientific notation.

   2.43×10^{11}

2. Write the number 3.6×10^{12} in standard notation.

   3,600,000,000,000

3. Explain the advantage of writing numbers in scientific notation.

   Writing numbers in scientific notation makes it possible to write very large or very small numbers quickly and makes them easier to read.

4. Explain the process for writing numbers in scientific notation.

   Answers will vary, for example: "When writing a very large number, break the number into two parts. First move the decimal point so that there is one digit to the left of it. Next, multiply that number by a power of 10 that is determined by the number of spaces that the decimal point was moved.

✱ SOL 7.1, 8.1

Prerequisite SOL
5.1

Lesson Summary
Students discover patterns for converting very large and very small numbers into scientific notation. They determine equivalent relationships between numbers written in standard notation and numbers written in scientific notation. (45 minutes)

Materials
"Number Notation Table" handouts
"Planet Distance Table" handouts
Computers with Internet access

Vocabulary
scientific notation. A system for writing very large or very small numbers as a number between 1 and 10 multiplied by a power of 10.

Warm-up
1. Write the following on the board: "There are approximately 6,000,000,000 people on Earth. Can you explain how many people this is? Do you think there is room for 6,000,000,000 more people?" Allow students to share their thoughts.
2. Tell students to assume that every person on Earth has 10 fingers and 10 toes. Have students figure out how many human fingers and toes there are. (20 x 6,000,000,000 = 120,000,000,000) Allow students to share their answers and the techniques they used to arrive at their answers to see if any students solved the problem in a way other than standard multiplication — e.g., multiply 6 times 20 and add 9 zeros. Ask, "Why might you add 9 zeros? What does adding 9 zeros mean?"
3. Distribute the "Number Notation Table" handouts. Lead the class in completing the first few rows of the table together. Discuss the pattern that emerges.
4. Have the students finish the table individually or in groups. Reinforce to the students that they should rely on the pattern just discussed. Review the answers when everyone is finished.

Lesson
1. Give each student a copy of the "Planet Distance Table" handout. Demonstrate how to change the distance Mercury is from the sun, 35,000,000 miles, into scientific notation. (3.5×10^7)
2. Discuss how the number was changed, and have students compare the pattern they discovered in the warm-up to the number. Ask students how the exponent is related to the decimal shift. Students may assume that the exponent number equals the number of zeroes; however, the exponent number (power of ten) equals the number of places the decimal moves, e.g., $1,400 = 1.4 \times 10^3$: the decimal moves three places to the left. Ask, "Why does the decimal move three places to the left?" Demonstrate this decimal move by multiplying 1.4 x 1,000 on the board.
3. Demonstrate on the board the process in the previous step. Multiply 3.6 x 100, 3.6 x 1000, and 3.6 x 10. Show all steps. Let the students discover how the decimal point moves. Discuss whether your number is greater than or less than 1. Change 3.5×10^7 back to standard form.
4. Compare 1×10^7 to 3.5×10^7. Emphasize again that <u>the exponent number (power of ten) equals the number of places the decimal moves.</u>
5. Explain to students that scientific notation can also be used to write very small numbers. Write 0.0000046 on the board. Review the steps below:
   - <u>Determine the new number between 1 and 10</u>: Move the decimal point in the original number so that the new number is between 1 and 10. (4.6)

- Determine the power of 10: Count the number of places you moved the decimal point. (–6) This number equals the exponent. Write the power of 10 that you would need to multiply the new number by in order to get the original number. (10^{-6})
- Write the two parts as a multiplication expression. (4.6×10^{-6})
- Demonstrate how multiplication of 4.6 x 0.000001 = 0.0000046.

6. Have students complete the "Planet Distance Table," and then review answers. Stress that the sign of the exponent in a power of 10 tells whether the number is less than or greater than 1. Have students write the following rules on their scientific notation handouts:
   - A power of 10 with a **positive exponent,** such as 10^5, means the decimal is greater than 1.
   - A power of 10 with a **negative exponent,** such as 10^{-5}, means the decimal is less than 1.
7. Have students go to http://www.aaamath.com. Seventh-grade students should go to the seventh-grade topics; eighth-grade students should go to the eighth-grade topics. Under each grade's topics, have students choose "Scientific Notation" and do 10 to 20 exercises from the following topics:
   - Converting Numbers to Scientific Notation
   - Converting Scientific Notation to Numbers
   - Comparing Scientific Notation and Standard Numbers
   - Converting Decimals to Scientific Notation
   - Converting Scientific Notation to Decimals

Reflection

Conduct a class discussion around the following questions:

- Why is scientific notation used?
- What are some careers or professions is which scientific notation is regularly used?
- In what professions would very large numbers be used?
- In what professions would very small numbers be used?

Name: ______________________

Number Notation Table

Number Spelled Out	Number in Standard Notation	Number in Scientific Notation
One		
Ten		
One hundred		
One thousand		
Ten thousand		
One hundred thousand		
One million		
Ten million		
One hundred million		
One billion		
Ten billion		
One hundred billion		
One trillion		

Name: ANSWER KEY

Number Notation Table

Number Spelled Out	Number in Standard Notation	Number in Scientific Notation
One	1	1×10^{0}
Ten	10	1×10^{1}
One hundred	100	1×10^{2}
One thousand	1,000	1×10^{3}
Ten thousand	10,000	1×10^{4}
One hundred thousand	100,000	1×10^{5}
One million	1,000,000	1×10^{6}
Ten million	10,000,000	1×10^{7}
One hundred million	100,000,000	1×10^{8}
One billion	1,000,000,000	1×10^{9}
Ten billion	10,000,000,000	1×10^{10}
One hundred billion	100,000,000,000	1×10^{11}
One trillion	1,000,000,000,000	1×10^{12}

Name: ______________________

Planet Distance Table

Complete the table below.

Planet	Miles from the Sun in Standard Notation	Miles from the Sun in Scientific Notation
Mercury	35,000,000	
Venus		6.5×10^7
Earth		9.3×10^7
Mars	137,000,000	
Jupiter	467,000,000	
Saturn	850,000,000	
Uranus		1.7×10^9
Neptune		2.7×10^9
(dwarf planet) Pluto	3,500,000,000	

Name: ANSWER KEY

Planet Distance Table

Complete the table below.

Planet	Miles from the Sun in Standard Notation	Miles from the Sun in Scientific Notation
Mercury	35,000,000	3.5×10^7
Venus	65,000,000	6.5×10^7
Earth	93,000,000	9.3×10^7
Mars	137,000,000	1.37×10^8
Jupiter	467,000,000	4.67×10^8
Saturn	850,000,000	8.5×10^8
Uranus	1,700,000,000	1.7×10^9
Neptune	2,700,000,000	2.7×10^9
(dwarf planet) Pluto	3,500,000,000	3.5×10^9

✱ SOL 7.12

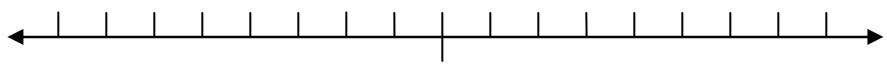
Prerequisite SOL
6.5

Lesson Summary
Students identify and graph ordered pairs in the four quadrants of a coordinate plane. (50 minutes)

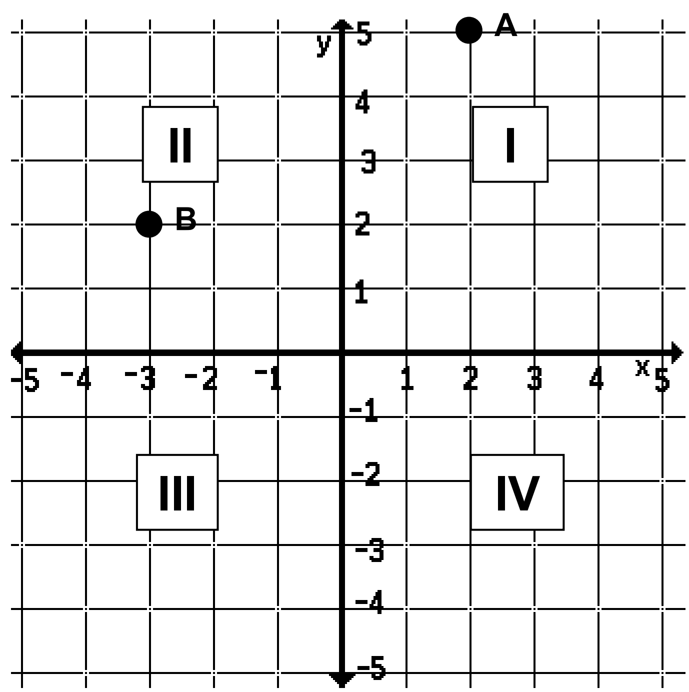
Materials
Graph paper
"City Map" handouts
"Coordinate Plane Practice" worksheets

Vocabulary
quadrants. The four sections of the coordinate plane determined by the *x*-axis and the *y*-axis.

Warm-up
Draw a number line on the board and label the zero point. Ask students to copy it onto a piece of paper, and then ask them to add the points 4, 6, –7, 2, –1, 8, –4. When everyone has completed their own line, have students come to the board to add the points. Tell the students that this number line is like the *x*-axis of a coordinate plane.



Lesson
1. Distribute a copy of the "City Map" handout to each student. Compare the coordinate plane to the city map: the *x*-axis and *y*-axis are like the two main city streets, and the point where these two main streets cross is the origin. Ask students to find the two main city streets on the map. (Main Street and Palm Blvd.) Have students describe the location of each building by using the names of the intersecting streets on which each is found — the street that runs horizontally first and then the street that runs vertically. (Library: Oak Street and Maple Street; Church: Elm Street and Pine Street: Pharmacy: Main Street and Palm Blvd.)
2. Explain that just as a place on the map can be located by looking for the intersection of two streets, a point on a coordinate plane can be located by finding the intersection of two lines. The horizontal lines on a coordinate plane are numbered using positive numbers from the origin to the right and negative numbers from the origin to the left. The vertical lines are numbered using positive numbers from the origin up and negative numbers from the origin down. Notice that each axis has arrows on both ends to show that they continue into infinity. The horizontal line is called the *x*-axis and the vertical line is called the *y*-axis. Discuss these points with the students while drawing



a coordinate plane in large graph paper or graph paper projected on the board. Have each student draw a coordinate plane on graph paper and label.

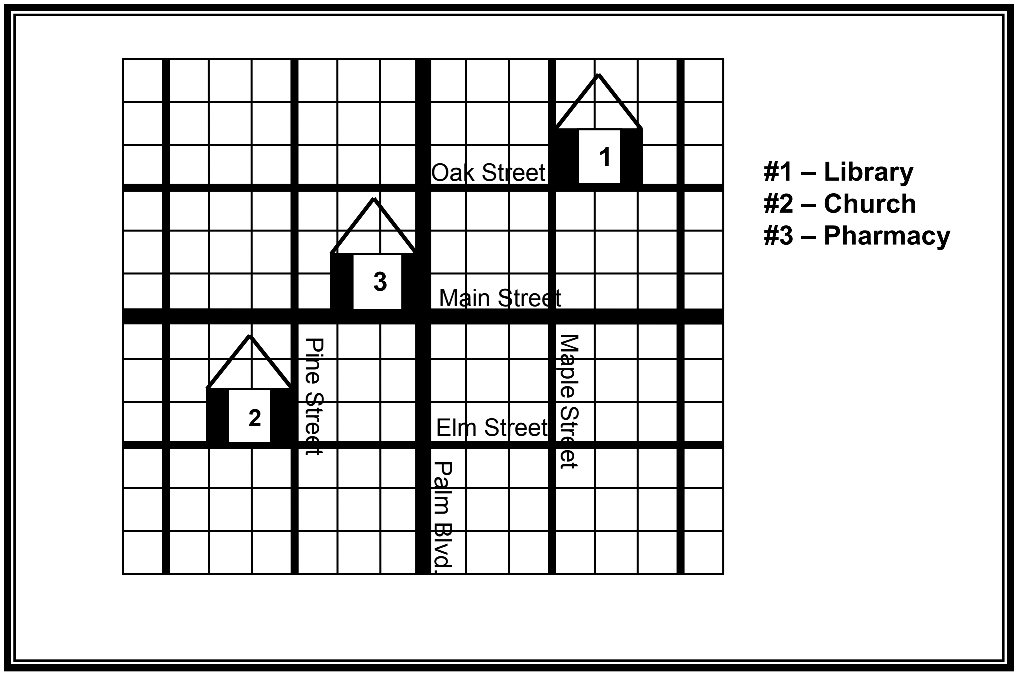
3. Point out that the *x*-axis and *y*-axis divide the coordinate plane into four sections called *quadrants.* These quadrants are numbered counterclockwise from 1 to 4, using Roman numerals and beginning in the upper right quadrant. Have students number the quadrants on their coordinate planes.
4. Inform students that a point located on a coordinate plane is written as a pair of numbers called *ordered pairs.* The first number of the pair is the *x*-coordinate and is found on the *x*-axis; the second number of the pair is the *y*-coordinate and is found on the *y*-axis. Ordered pairs are written in parentheses separated by a comma. For example, if you are asked to locate the ordered pair (2, 5), you would start at the origin and move 2 spaces to the right on the *x*-axis and then 5 spaces up. The intersection would be labeled with a letter, A, to represent the location of ordered pair (2, 5). You would say that ordered pair (2, 5) is at point A (see coordinate plane on previous page).
5. Explain that when the *x*-coordinate is a negative number, you move to the left of the origin on the *x*-axis and then up or down to the *y*-axis. Ask students to locate ordered pair (–3, 2) and label it point B (see coordinate plane on previous page).
6. Distribute the “Coordinate Plane Practice” worksheet, and have students complete it. Assist students individually who had difficulty finding the location of (–3, 2) in the previous step.

Reflection

Have students describe in writing how locating a point on the coordinate plane is like finding a building on a city map.

Name: ________________________

City Map



The library is located at the intersection of ________________________________.

The church is located at the intersection of ________________________________.

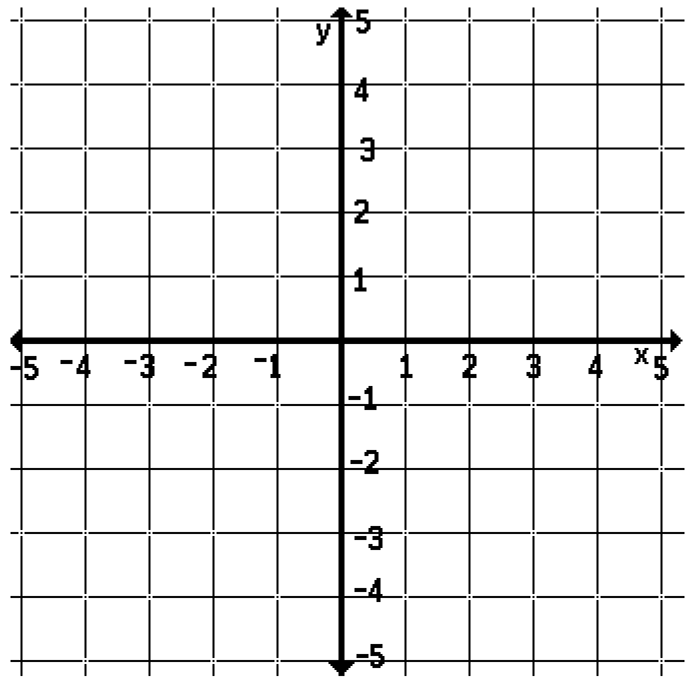
The pharmacy is located at the intersection of ________________________________.

Name: ________________________

Coordinate Plane Practice

Locate the following ordered pairs on the coordinate plane at right, and label each point with the appropriate letter.

1. A (2, 3)
2. B (0, –2)
3. C (–2, 4)
4. D (0, 2)
5. E (5, –1)
6. H (–1, –4)
7. J (3, –3)
8. K (1, 0)



Use Roman numerals to name the quadrant in which each point is found. If a point is located on an axis, identify the axis on which it lies.

A ______

B ______

C ______

D ______

E ______

H ______

J ______

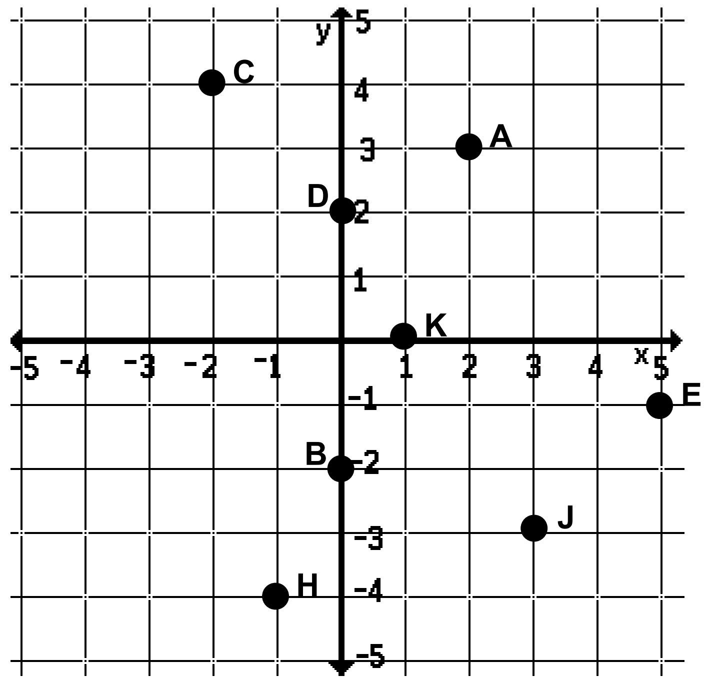
K ______

Name: <u>ANSWER KEY</u>

Coordinate Plane Practice

Locate the following ordered pairs on the coordinate plane at right, and label each point with the appropriate letter.

1. A (2, 3)
2. B (0, –2)
3. C (–2, 4)
4. D (0, 2)
5. E (5, –1)
6. H (–1, –4)
7. J (3, –3)
8. K (1, 0)



Use Roman numerals to name the quadrant in which each point is found. If a point is located on an axis, identify the axis on which it lies.

A <u>I</u>

B <u>on the *y*-axis</u>

C <u>II</u>

D <u>on the *y*-axis</u>

E <u>IV</u>

H <u>III</u>

J <u>IV</u>

K <u>on the *x*-axis</u>

✱ SOL 7.19

Prerequisite SOL
5.20, 6.21

Lesson Summary
Students identify, distinguish between, explain, and extend simple arithmetic and geometric sequences. (50 minutes)

Materials
Scientific calculators
"Warm-up" worksheets
"Mr. Jones' Canned Goods Display" worksheets
"Sequences Practice" worksheets

Vocabulary
arithmetic sequence. A series of numbers such that each term is equal to the one before it plus some constant number.
geometric sequence. A series of numbers such that each term is equal to the one before it multiplied by a constant number.
terms. The numbers in a sequence.

Warm-up
Distribute the "Warm-up" worksheet, and have students work in collaborative groups to find the next three terms in each sequence. Each group should appoint a spokesperson to explain the rule used to find the terms. Once each group completes the assignment, have the spokespersons report to the whole class.

Lesson
1. Go back to the three sequences used in the warm-up. For sequence #1, review student answers, and explain that when each term in a sequence equals the term before it plus some term, it is called an *arithmetic sequence.* Have the class create an arithmetic sequence.
2. For sequence #2, review student answers. Explain that when each term in a sequence equals the term before it multiplied by a certain number, it is called a *geometric sequence.* Work with the class to create a geometric sequence.
3. For sequence #3, review student answers. Explain that because the terms in this sequence differ by one, but sometimes 1 is being added and other times 1 is being subtracted, the sequence is *not* arithmetic. Also, because there is no common value being multiplied by each term to get the next, the sequence is *not* geometric. Help the class develop a few examples of sequences that are not arithmetic nor geometric.
4. Explain that patterns can also be written in table form. Use following problem: Jake is trying out for the cross country team. He is setting up a practice schedule to increase his endurance.

Day	1	2	3	4	5
Minutes	15	17	20	24	29

   Ask the students to describe the pattern. (Each day the increase is 1 more minute than the previous day's increase: +2,+3,+4, +5, and so on.) Assuming the pattern continues, how many minutes would he plan to run on day 6? (35 minutes) Is this pattern arithmetic, geometric, or neither? (Neither) Why? (The same number is not being added to each previous term, nor is each term being multiplied by a certain value.)
5. Explain that patterns can also be shown in graphic form. Pass out the "Mr. Jones' Canned Goods Display" worksheets, and have the students answer the questions.

6. Introduce the "Sequences Practice" worksheets. When all students have completed the practice, review the answers with them.

Reflection

Put the following sequence on the board: 4, 6, 12, 14, 28, 30, 60, . . ., and ask the students the following questions about it:

- How could you explain this sequence in words: (The pattern is the previous term + 2, then the previous term x 2, then the previous term + 2, then the previous term x 2, . . .).
- What are the next two terms in this pattern? (62, 124)
- Is this sequence arithmetic, geometric, or neither? (Neither)
- Why? (This sequence is neither arithmetic nor geometric because the same number is not being added to each previous term, nor is each term being multiplied by the same number.)

Name: ________________________

Warm-up

Find the pattern in each sequence below. What would the next three numbers be in each sequence? Write a short explanation for the rule for each.

1. 15, 30, 45, 60, ___, ___, ___ . . .

   Rule: ____

2. 4, 12, 36, 108, ___, ___, ___ . . .

   Rule: ____

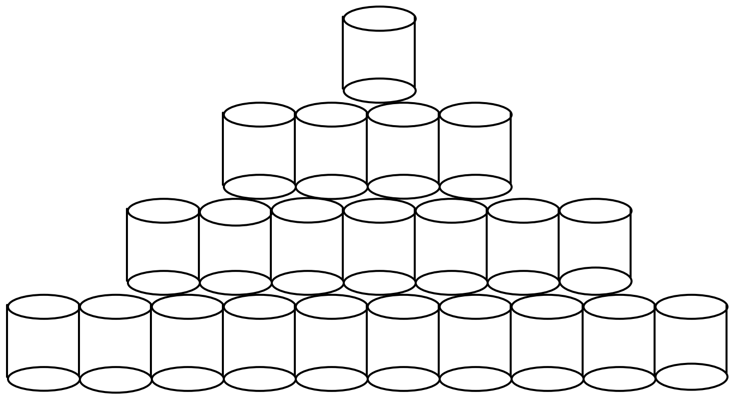
3. 1, 2, 3, 2, 1, 2, ___, ___, ___ . . .

   Rule: ____

Name: ______________________

Mr. Jones' Canned Goods Display

Mr. Jones has made a display of canned goods in his grocery store. The table below shows how many cans are in each row:



Row number	1	2	3	4
Number of Cans in Row	1	4	7	10

1. If each row has 3 more cans than the previous row, how many cans will be in the 10th row?

   ______

2. Is this pattern arithmetic, geometric, or neither? ___________________

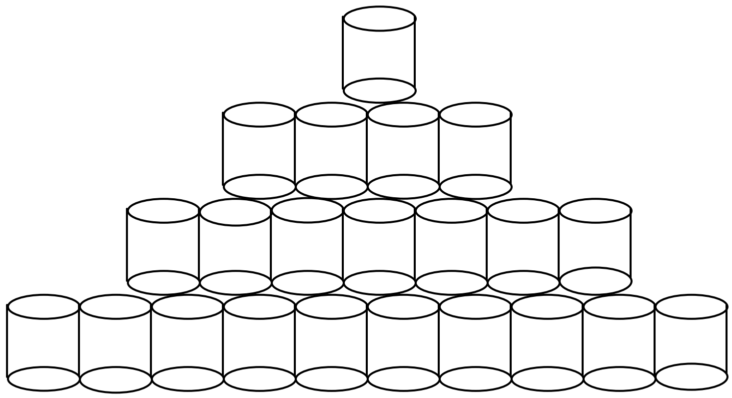
3. How do you know?

   ____

Name: <u>ANSWER KEY</u>

Mr. Jones' Canned Goods Display

Mr. Jones has made a display of canned goods in his grocery store. The table below shows how many cans are in each row:



Row number	**1**	**2**	**3**	**4**
Number of Cans in Row	**1**	**4**	**7**	**10**

1. If each row has 3 more cans than the previous row, how many cans will be in the 10^{th} row?

    __28__

2. Is this pattern arithmetic, geometric, or neither? ___Arithmetic_____

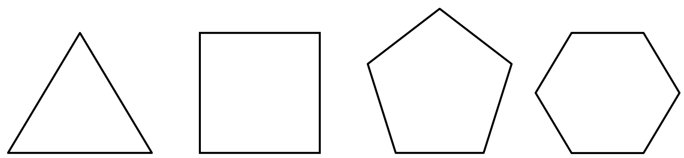
3. How do you know?

    Because the same number, 3, is being added to each previous term.

Name: ________________________

Sequences Practice

Find the next two terms (numbers or graphics) in each sequence. Describe each sequence as arithmetic, geometric, or neither. Explain your reason.

1. 

2. 4, 11, 18, 25, 32, ___, ___

3. 1, 4, 8, 13, 19, ___, ___

4. 1, 3, 9, 27, ___, ___

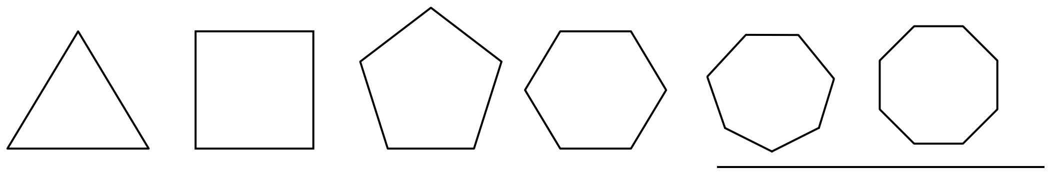
5. 0.2, 0.4, 0.8, 1.6, ___, ___

6. 1, 4, 9, 16, 25, ___, ___

Name: ANSWER KEY

Sequences Practice

Find the next two terms (numbers or graphics) in each sequence. Describe each sequence as arithmetic, geometric, or neither. Explain your reason.

1. 

   arithmetic — Add 1 side to each term.

2. 4, 11, 18, 25, 32, 39, 46 — arithmetic — Add 7 to each term.

3. 1, 4, 8, 13, 19, 26, 34 — neither — Add +3, +4, +5, ….

4. 1, 3, 9, 27, 81, 243 — geometric — Multiply each term by 3.

5. 0.2, 0.4, 0.8, 1.6, 3.2, 6.4 — geometric — Multiply each term by 2.

6. 1, 4, 9, 16, 25, 36, 49 — neither — Add +3, +5, +7, ...

✱ SOL 7.20

Prerequisite SOL
5.21, 6.23

Lesson Summary
Students write verbal expressions as algebraic expressions. (50 minutes)

Materials
White boards	“Warm-up” worksheets
Dry-erase markers	“Vocabulary Chart” worksheets

Vocabulary
expression. A combination of numbers and/or variables using mathematical operations and no equal sign.
coefficient. The constant factor of a term.
variable. A symbol, usually a letter, representing an unknown quantity.

Warm-up
Distribute the “Warm-up” worksheets, and allow students to work in pairs to complete the chart.

Lesson
1. Ask the class whether anyone speaks a foreign language. Ask what it means to *translate* one language to another. Explain to the students that in this lesson, they will be *translating* English phrases into algebraic expressions.
2. Put the students into groups. Distribute the “Vocabulary Chart” worksheets, and ask students to work in groups to complete it.
3. Review and discuss responses, and add any additional words the students may generate.
4. Using the completed vocabulary chart, walk the students through translating the following verbal expressions:
   a. **Three runs more than the Yankees scored.** “Do we know how many runs the Yankees scored?” (No) “So, let r represent the number of runs the Yankees scored. The words ‘more than’ suggest addition, as shown on the chart. So, the expression should be $r + 3$.”
   b. **Twice as many tomatoes as last year.** “Do we know how many tomatoes were grown last year?” (No) “So, let t represent the number of tomatoes grown last year. The word “twice” suggests multiplication by 2, as shown on the chart. So, the expression should be $2t$.”
   c. **Half as many apples as John picked.** “Do we know how many apples John picked?” (No) “So, let a represent the number of apples John picked. The word “half” suggests division by 2, as shown on the chart. So, the expression should be $a \div 2$ or $\frac{a}{2}$.
   d. **Seven fewer fireflies than Tammy caught.** “Do we know how many fireflies Tammy has? (No) “So, let f represent the number of fireflies that Tammy has. The word “fewer” suggests subtraction, as shown on the chart. So the expression should be $f - 7$.
5. Distribute a white board to each group of students. Present each of the following phrases, one at a time, to the students. Students should discuss the phrase with their group before writing an algebraic expression for it on the board. Remind groups to conceal their answers until you call for them.
   a. The difference between g and 4 ($g - 4$, or $4 - g$)
   b. The quotient of b and 5 ($b \div 5$ or $\frac{b}{5}$, or $5 \div b$ or $\frac{5}{b}$)
   c. Seventeen less than p ($p - 17$)
   d. Five years older than Paul ($p + 5$)

e. Jamila's salary plus $1,100 ($s$ + $1,100)

f. The product of x and 2 ($x \cdot 2$ or $2x$)

Reflection

Have the students translate the following phrases into algebraic expressions and then write a short explanation of how they know their expression is correct.

1. Nine increased by some number. ("Increased by" suggests addition. Let x represent the unknown number. So, the algebraic expression should be $9 + x$.)
2. The sum of a number and six. ("Sum" suggests addition. Let p represent the unknown number. The algebraic expression should be $p + 6$.)
3. Twice as many apples. ("Twice" suggests multiplication by 2. Let a represent the number of apples. So, the algebraic expression should be 2 x a or $2 \cdot a$ or $2a$.)

Name: ________________________

Warm-Up

Algebraic Expression	Variables	Coefficients	Operations
$4x - 2y$			
$3r - 2s + 5t$			
$48a \div b$			
$21 \div 3p$			

Name: ANSWER KEY

Warm-Up

Algebraic Expression	Variables	Coefficients	Operations
$4x - 2y$	*x, y*	4, 2	multiply, subtract
$3r - 2s + 5t$	*r, s, t*	3, 2, 5	multiply, subtract, add
$48a \div b$	*a, b*	48, 1	multiply, divide
$21 \div 3p$	*p*	3	multiply, divide

Name: ______________________

Vocabulary Chart

Work with your group to place the following words/phrases under the appropriate headings.

times	**less**	**sum**	**divided**	**more than**
plus	**multiplied**	**quotient**	**total**	**decreased by**
of	**minus**	**increased by**	**product**	**twice**
difference	**less than**	**half**	**fewer**	**fewer than**

Addition	Subtraction	Multiplication	Division

Name: ANSWER KEY

Vocabulary Chart

Work with your group to place the following words/phrases under the appropriate headings.

times	**less**	**sum**	**divided**	**more than**
plus	**multiplied**	**quotient**	**total**	**decreased by**
of	**minus**	**increased by**	**product**	**twice**
difference	**less than**	**half**	**fewer**	**fewer than**

Addition	Subtraction	Multiplication	Division
plus	minus	times	divided
sum	difference	product	quotient
more than	less than	multiplied	
increased by	less	of	
total	decreased by	twice	
	fewer		
	fewer than		

✱ SOL 7.22

Prerequisite SOL
6.5, 7.5, 7.20, 7.21

Lesson Summary
Students solve one-step linear equations in one variable. (Part one of a two-part lesson: 50 minutes)

Materials
Cups
Counters
“Equation Mat Practice 1” worksheets

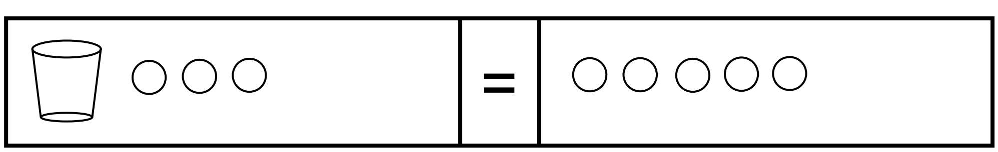
Vocabulary
equation. A mathematical sentence stating that two expressions are equal.
coefficient. The constant factor of a term.
variable. A symbol, usually a letter, representing an unknown quantity.

Warm-up
Write the following equations on the board, and ask students to complete them:

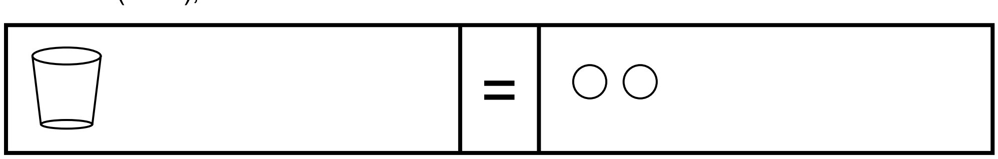
1. $7 + \square = 16$ 2. $18 - 4 = \square$ 3. $\square \bullet 7 = 21$ 4. $\square \div 6 = 9$

Allow students three to five minutes to solve the equations. Then, ask students whether they realized that when they do these types of problems, which they have done for years, they are actually doing algebra. Tell them that the difference is that in an algebraic equation, the box is replaced with a variable, which is any lower case letter that is used to represent the unknown number. For example, the first equation would be written algebraically as $7 + x = 16$. Have the students write the other three equations, using a variable instead of the box.

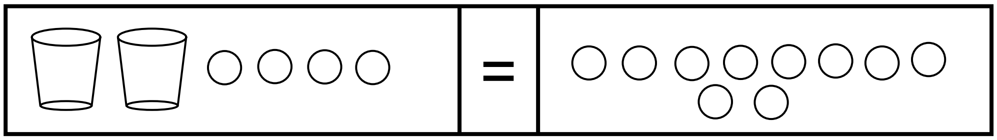
Lesson
1. Tell students that one way to visualize and understand an algebraic equation is by using models, such as cups, counters, and an “equation mat.” Explain that the cup represents the unknown value or variable (x) in the equation, and each counter represents 1.
2. Model the algebraic equation $x + 3 = 5$ for the students by placing 1 cup (x) and 3 counters on the left side of an equation mat and placing 5 counters on the other right of the mat. Tell the student that this equation on the mat is *balanced*. That means that the combination of the 1 cup and 3 counters <u>is the same as</u> or <u>is equal to</u> 5 counters.



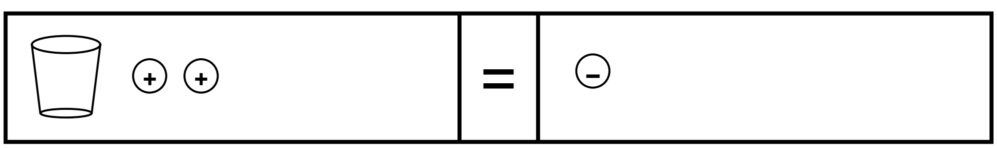
3. Remind the students that the goal is to find the value of the cup. To do this, we need to get the cup by itself on one side of the equation mat and still have the equation in balance. To get the cup by itself, we remove the 3 counters from the left side, and to keep the equation balanced, we must also remove 3 counters from the right side of the equal sign. We then can see that the cup equals 2 counters (ones), or $x = 2$.



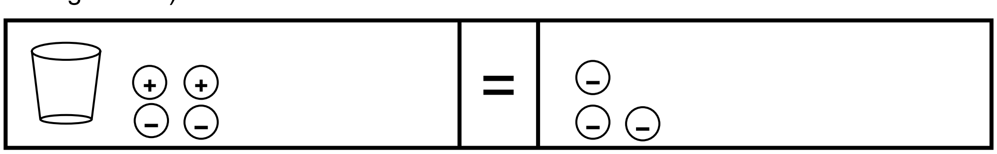
4. Distribute the “Equation Mat Practice 1” worksheets, cups, and counters, and direct the students to solve equations 1, 2 and 5, using the manipulatives on the mat. Allow students to work in partners, and provide assistance to partners as needed.
5. If time is an issue, you may begin your next lesson with this step. Explain to the students that up to this point, they have been solving equations with only addition and subtraction. The equation $2x + 4 = 10$ involves some multiplication. Ask the class what the term “$2x$” means. (“Two x’s” or, in our model, two cups.) Then, set up the model of the equation by adding the 4 counters next to the 2 cups on the left side of the mat and putting 10 counters on the right side to form a balanced equation.



6. Ask the students to tell you the next step in solving this equation, using the model. (Remove 4 counters from each side of the equation, thus leaving 2 cups equal to 6 counters [ones]). Because 2 cups are equal to 6 counters, 1 cup must equal 3 counters.
7. If students need additional practice as a group, use the equation $3x + 2 = 8$, and model it with the procedure outlined in step 5.
8. When the students are ready for individual practice, assign equations 3, 4, and 6 on the worksheet.
9. Explain to the students that so far, they have been solving equations with only positive numbers, but now they will see how to solve equations that include negative numbers. Display 3 positive counters (3 ones) to the students, and ask them the value of the 3 counters. Then, place 3 negative counters (minus ones) next to the 3 positive counters, and ask the students the value of the 6 counters all together. They should respond by saying “zero.” If they do not, be sure to place each positive counter directly next to a negative counter so they can see why the value is zero. Refer to the combination of a positive and a negative together as a “zero pair.” Ask the students to make zero in any form with their counters. Some will simply use 1 positive and 1 negative counter, while others may use 5 (or any equal number) of each. Allow students to share their zero pairs with each other.
10. Tell the students they will now use zero pairs to help them solve equations like this one: $x + 2 = –1$. Model this equation by placing 1 cup and 2 positive counters (ones) on the left side of the equation mat. Place 1 negative counter (minus one) on the right side of the mat. Remind the students that the goal is to get the cup by itself on one side of the equation mat.

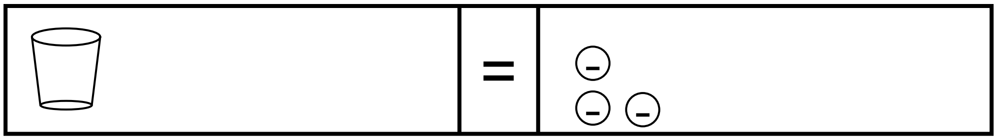


11. Ask the students whether it is possible to remove 2 positive counters from each side of the mat. (No, since there are no positive counters on the right.) Ask them what they could do to “cancel out” or reduce to zero the 2 positive counters on the left. (Add 2 negative counters to the left side to create 2 zero pairs on that side.) Ask, “If you add 2 negative counters to the left side, must you do anything to the right side to keep the equation balanced?” (Yes, add the same thing — 2 negative counters — on the right side.)



12. Point out that you now have 2 zero pairs on the left side and that they can simply be removed because they are equal to zero. Ask, “Does removing these 2 zero pairs change the value of the left side of the mat?” (No, because you are removing only zero.) “Must you do anything to the right side to keep the equation balanced?” (No) Point out that you have now solved the equation: the cup is now

by itself on one side of the mat, and there are 3 negative counters (minus ones) on the other side. Therefore, the cup is equal to 3 minus ones, or $x = (-3)$



13. Direct students to complete the practice worksheet by solving equations 7, 8 and 9, and provide partners to groups as needed.

Reflection

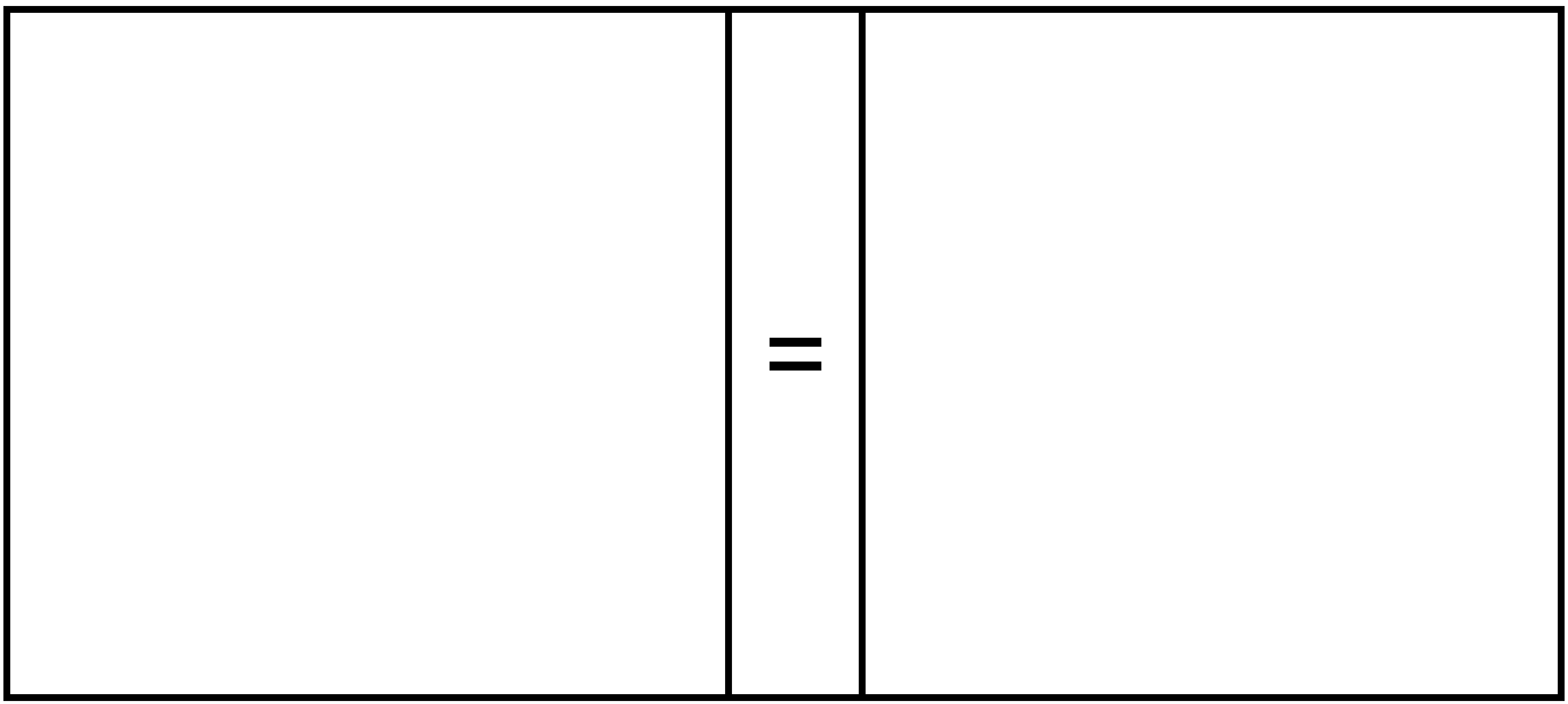
Have students explain the similarity between solving algebraic equations and solving arithmetic problems that they did in elementary grades. (A variable instead of an empty box is used to represent the unknown amount.)

Name: ______________________

Equation Mat Practice 1

Solve the following equations, using cups and counters on the equation mat shown below:

1.	$x + 1 = 6$	2.	$x + 2 = 5$	3.	$5x = 10$
4.	$4x = 16$	5.	$x + 0 = 3$	6.	$3x = 9$
7.	$r + 3 = (–9)$	8.	$m + 2 = (–4)$	9.	$n + 4 = (–1)$



✱ SOL 7.22

Prerequisite SOL

6.5, 7.5, 7.20, 7.21

Lesson Summary

Students solve one-step linear equations in one variable. (Part two of a two-part lesson: 50 minutes)

Materials

Cups	"Warm-up" worksheets
Counters	"Equation Mat Practice 2" worksheets

Vocabulary

equation. A mathematical sentence stating that two expressions are equal.
coefficient. The constant factor of a term.
variable. A symbol, usually a letter, representing an unknown quantity.

Warm-up

Distribute the "Warm-up" worksheets, cups, and counters. Direct students to solve the equations. Provide assistance to individuals as needed.

Lesson

1. Review by saying, "Think of an equation as a balance or scale. The expression to the left of the equal sign should be equal to the expression on the right side of the equal sign. If the two parts are not equal, the scale will not balance."
2. Model the equation $x + 7 = 10$ with cups and counters on the equation mat, while simultaneously translating your moves with the manipulatives into algebraic symbols. For example, as you remove 7 chips from both sides of the equation mat, demonstrate this as –7 from the left and right sides of the equal sign.

   $$\begin{aligned} x + 7 &= 10 \\ x + 7 - 7 &= 10 - 7 \\ x &= 3 \end{aligned}$$

   Check the answer by placing it in the original equation: $3 + 7 = 10$, or $10 = 10$.
3. Point out to students that if you subtract 7 only from the left side, your equation becomes unbalanced.

   $$\begin{aligned} x + 7 &= 10 \\ x + 7 - 7 &= 10 \\ x &= 10 \end{aligned}$$

   Check this answer by placing it in the original equation: $10 + 7 = 10$, or $17 \neq 10$.
4. Distribute the "Equation Mat Practice 2" worksheets, and direct students to complete equations 1 and 4. Provide assistance where needed.
5. Model the equation $x - 4 = 5$ with cups and counters on the equation mat, using zero pairs. Simultaneously translate your moves with the manipulatives into algebraic symbols.

   $$\begin{aligned} x - 4 &= 5 \\ x - 4 + 4 &= 5 + 4 \\ x &= 9 \end{aligned}$$

   Check the answer by placing it in the original equation: $9 - 4 = 5$, or $5 = 5$.
6. Direct students to complete the "Equation Mat Practice 2" worksheet. Provide assistance where needed.

Reflection

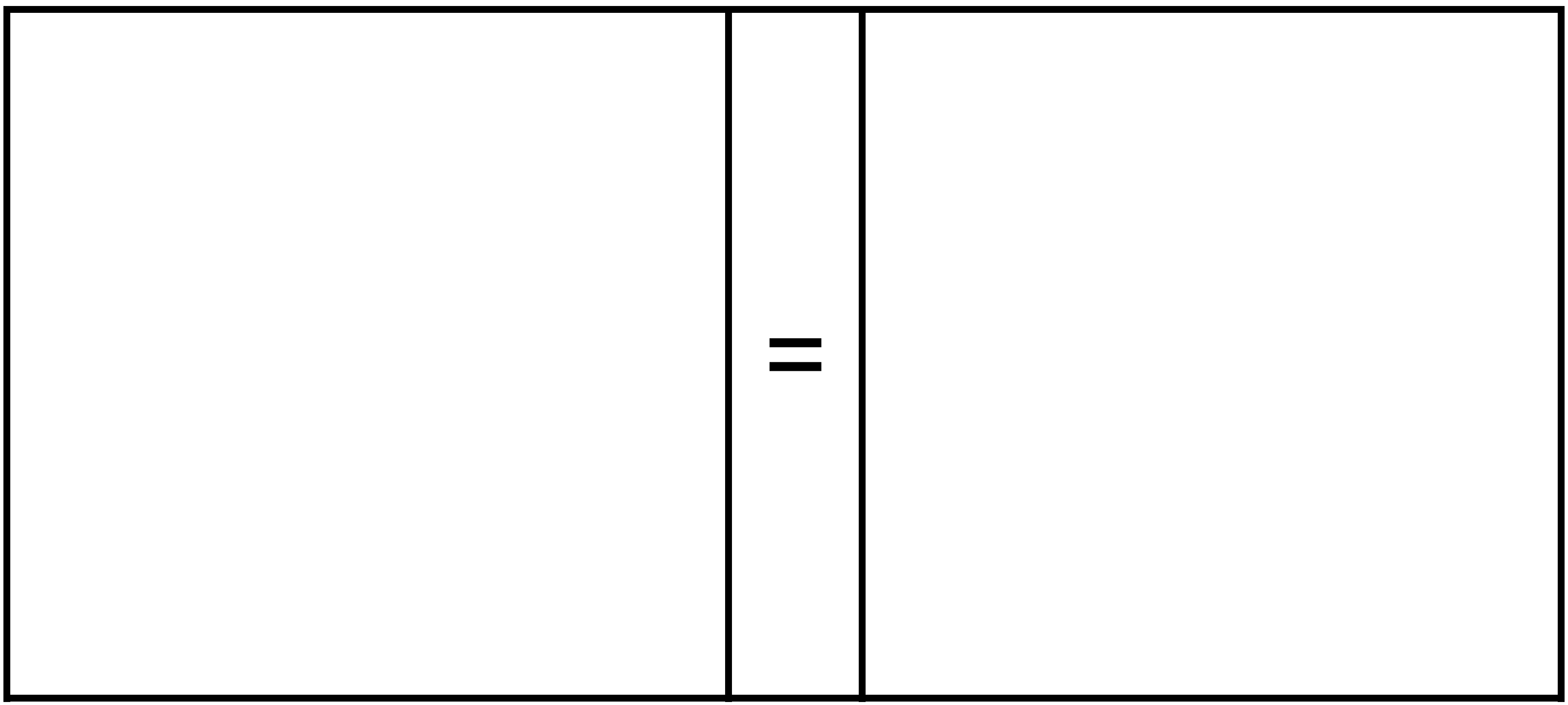
Have students describe in writing how an algebraic equation is like a balance and give an example of an equation that does balance.

Name: ______________________

Warm-up

Solve the following equations, using cups and counters on the equation mat shown below:

1. $x + 5 = 10$
2. $x + 7 = 10$
3. $y + 2 = 5$
4. $t + 8 = 9$
5. $a + 4 = 6$
6. $x + 6 = 11$

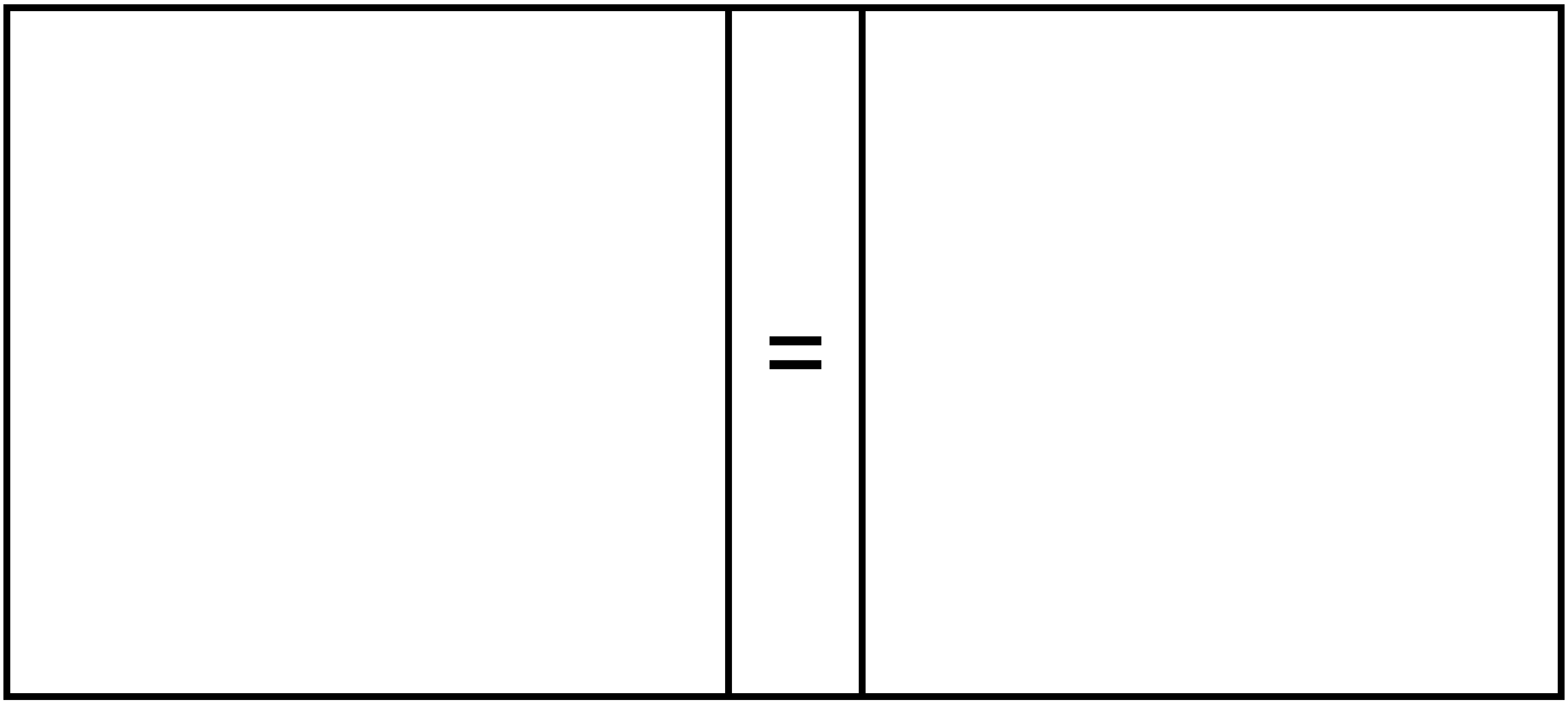


Name: ________________________

Equation Mat Practice 2

Solve the following equations, using cups and counters on the equation mat shown below. Show a check for each.

1.	$a + 3 = 12$	2.	$g – 6 = 4$	3.	$b – 8 = 5$
4.	$h + 7 = 15$	5.	$c – 9 = 2$	6.	$s + 6 = 15$
7.	$d + 5 = 9$	8.	$x – 3 = 17$		



✱ SOL 7.22

Prerequisite SOL
6.5, 7.5, 7.20, 7.21

Lesson Summary
Students solve one-step linear equations in one variable. (50 minutes)

Materials
"Warm-up" worksheets
"Algebra Practice" worksheets

Vocabulary
equation. A mathematical sentence stating that two expressions are equal.
variable. A symbol, usually a letter, representing an unknown quantity.

Warm-up
Distribute the "Warm-up" worksheets, and ask students to solve the equations. Go over answers and discuss the students' work.

Lesson
1. Ask a student to read the following problem aloud to the group: **The Yankees and the Red Sox played a baseball game. The Yankees scored 5 runs. The sum of the two scores was 12. What was the Red Sox' score?** Explain that the first step in solving the problem is choosing a variable to represent the unknown, e.g., the Red Sox' score. Write "r = the Red Sox' score" on the board for reference. Then, write an algebraic equation for the problem: $5 + r = 12$, which means "the Yankees' score of 5 runs plus the Red Sox' score, r, is 12." Solve this equation on the board with student input. State clearly in writing at the end that the Red Sox scored 7 points.
2. Ask a student to read the following problem aloud to the group: **John has 10 pairs of socks in his drawer. Some are dress socks and some are athletic socks. He has 3 pairs of dress socks. How many pairs of athletic socks are in his drawer?** Demonstrate that the first step in solving the problem is choosing a variable to represent the unknown, e.g., the number of pairs of athletic socks in the drawer. Write "w = the number of pairs of athletic socks in the drawer" on the board for reference. Then, write an algebraic equation for the problem: $w + 3 = 10$. Solve the equation with input from the students. State clearly in writing at the end that there are 7 pairs of athletic socks in the drawer.
3. Ask a student to read the following problem aloud: **Maggie picked up 4 times as many golf balls on the golf course as Ron. The total number of golf balls in Maggie's bucket is 36. How many golf balls did Ron find?** Let r represent the number of golf balls Ron found. Write "r = the number of golf balls Ron found" on the board for reference. Remember that the word *times* means "to multiply." Write an algebraic equation to solve the problem: $4r = 36$. Solve the equation with student input.
4. Distribute the "Algebra Practice" worksheets, and allow students to work with a partner to solve the problems and show their work. Assist students who have difficulty.

Reflection
Have students write a real-life problem that they can express and solve with an algebraic equation. Be sure they include the equation and its solution.

Name: ________________________

Warm-up

Solve the following algebraic equations. Show a check for each.

1. $x + 5 = 10$

2. $x - 7 = 10$

3. $x - 2 = 5$

4. $x + 8 = 9$

5. $x + 4 = 6$

6. $x - 6 = 11$

Name: ______________________

Algebra Practice

Write an algebraic equation for each problem, and solve the equation.

1. Wanda lost 4 pounds this week on her weight-loss program. If her present weight is 160 pounds, what was her weight when she started?

2. Terry mows 5 lawns every week. How much money must he make on each lawn job in order to make a total of $115 per week?

3. Tonya's math average is 8 points higher than Joe's. If Tonya has an 87 average in math, what is Joe's math average?

Name: ANSWER KEY

Algebra Practice

Write an algebraic equation for each problem, and solve the equation.

1. Wanda lost 4 pounds this week on her weight-loss program. If her present weight is 160 pounds, what was her weight when she started?

   Let w = Wanda's weight when she started

   $w - 4 = 160$

   $w = 164$

   Wanda weighed 164 pounds when she started.

2. Terry mows 5 lawns every week. How much money must he make on each lawn job in order to make a total of $115 per week?

   Let t = the amount of money Terry must make on each lawn job

   $5t = 115$

   $t = 23$

   Terry must make $23 on each lawn job in order to make a total of $115.00 per week.

3. Tonya's math average is 8 points higher than Joe's. If Tonya has an 87 average in math, what is Joe's math average?

   Let t = Joe's math average

   $t + 8 = 87$

   $t = 79$

   Joe's math average is 79.

✱ SOL 8.4

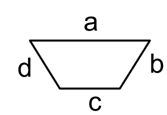
Prerequisite SOL
None

Lesson Summary
Students play a card game that provides practice in solving algebraic expressions, using order of operations and variables. Problems are limited to positive exponents. (60 minutes)

Materials
Decks of "expression cards" (templates attached)

Vocabulary
expression. A combination of numbers and/or variables using mathematical operations but no equal sign.
equation. A mathematical sentence stating that two expressions are equal.
variable. A symbol, usually a letter, representing an unknown quantity.

Warm-up
Have students use a variable to write an expression to represent each of the following sentences:
- If there are 5,280 feet in a mile, how many feet are in *m* miles? (5,280 x *m*)
- At Kings Dominion, all snow cones cost $1.00. How much would *s* cones cost? ($1.00 x *s*)
- At a local grocery store, bread costs $1.50 a loaf. How much would *b* loaves cost? ($1.50 x *b*)
- If there are 5 calories in a gram of protein and 10 calories in a gram of fat, use a variable expression to show the number of calories from fat and protein in any food. ($5p + 10f$)
- Write an expression to represent the perimeter of the trapezoid shown at right. ($a + b + c + d$)



Lesson
1. Put students into pairs, and distribute a deck of "expression cards" to each pair.
2. Tell the students to shuffle the cards and place the deck face down in the center of the table. Have each player select a card and place it face down in front of him/her.
3. Write $x = 2$ on the board. Explain to the students that when you say, "Go," they should turn their selected cards over and evaluate the expression shown on the card, using $x = 2$. Walk around the room and as students finish, give them a signal for a correct answer, such as a pat on the shoulder, a high five, or a "thumbs up."
4. When all expressions have been evaluated, have the pairs of students exchange cards. This time, write $x = 3$ on the board, and say "Go." Check answers as before.
5. After round two, have students select another expression card from the deck. Repeat rounds 1 and 2 until time is up or until all the cards have been used. Substitute any value for *x* that you deem appropriate for the expressions.
6. This exercise can become a game by giving partners a point for each correct answer.

Reflection
Have the class create their own deck of expression cards, and have students use these cards for additional practice.

a **$3x + 6$**	**b** **$2x + 4$**
c **$2(x - 1)$**	**d** **$4x + 2$**
e **$3x + 6$**	**f** **$3x + 3$**
g **$15 - (x + 7)$**	**h** **$-3x - 1$**

i $3x + 6$	**j** $\dfrac{x^3}{(3 \cdot 3)}$
k $5x(2^3 - 2)$	**l** $(-5^2)x$
m $2x + 6$	**n** $8^2 - x + 2$
o $x - (2 + 5)$	**p** $x^2 + 4$

q $3(x+6)$	**r** $3^3 + \frac{10}{x}$
s $11 - x^2 + 6$	**t** $4x + 7^2$
u $\frac{21}{(2x+1)}$	**v** $\frac{12}{(x+8)}$
w $-2x - \frac{12}{2}$	**x** $(-6)4x + 2$

✱ SOL 8.4, 8.14, 8.16, 8.17, 8.18

Prerequisite SOL

5.21, 6.23, 7.20, 7.12, 7.22

Lesson Summary

Students are introduced to real-world problems that involve writing, solving, and graphing two-step equations. (60 minutes)

Materials

"Warm-up" worksheets
"Using Equations in the Real World" worksheets
Calculators

Vocabulary

independent variable. The value that is manipulated or changed on purpose.
dependent variable. The value that responds to or is affected by the change.
domain. All possible input values of a function.
range. All possible output values of a function.

Warm-up

Distribute the "Warm-up" worksheets, and have the students complete it. Discuss their answers when they have finished.

Lesson

1. Begin the lesson by presenting the following situation: Maria delivers newspapers. She is paid $15.00 a day plus $.50 for each newspaper she delivers. How much money will Maria make if she delivers only 1 newspaper? ($15.50) 2 newspapers? ($16.00) 5 newspapers? ($17.50) Have students explain in words how they arrived at their answers. What operations did they use? (Multiplication and addition) Which operation did they use first? (Multiplication) Why? (Because the order of operations says to multiply or divide first and then to add or subtract.)
2. Ask for student volunteers to come to the board and translate their words into an equation. If necessary, prompt them by helping them decide what the variables will represent, i.e., m = money and n = newspapers. Lead students to derive the equation $.50$n$ + $15.00 = m, or **0.5*n* + 15 = *m***.
3. Tell students that this equation is an example of a pattern but that the pattern is hard to recognize by looking at the equation. Distribute the "Using Equations in the Real World" worksheets, and have students fill in the data table, which shows the pattern. Ask, "What is the pattern that the data in the table shows?" (A steady or even increase of $.50 per newspaper). Have students discuss the use of the pattern for finding any amount of money that Maria could make.
4. Once students understand how to use the equation, point out that Maria's earnings are *dependent* on the number of papers she sells. Therefore, m is called the *dependent variable.* On the other hand, n does not depend on any other value — it is independent — so it is called the *independent variable.*
5. Discuss how to graph the data in the table. Graph the first two points with the students, and allow them to complete the graph.
6. When the students have completed the graph, conduct a discussion. Have students name the three representations of the function that they have used (equation, data table, graph) and describe the use of each. (Using an equation allows for a fast calculation of any given value of the independent variable. A data table displays all of the data points. Looking at a graph displays the trend occurring based on the values of the independent variable.) Have students point out the merits of all three representations of the function.
7. It is important that you conduct a discussion of the graph. Students should *not* connect the points graphed. Ask them why. (Because the line between the whole number points would represent part of

a newspaper, and Maria does not deliver parts of newspapers!) Have a discussion about why connecting the points would show all possible answers to the equation but would not fit the word problem.

8. Discuss other word problems in which connecting the graphed points would or would not be appropriate, such as the following:
   - The amount of money made per hour worked
   - The amount of money charged for a speeding ticket based on the number of miles one is caught driving over the speed limit
   - The number of dolls sold and the amount of money made
9. At this point, ask students for the *domain* values (the *x*-axis values) in Maria's story and the *range* values (the *y*-axis values). Discuss how these are related to dependent and independent variables. (domain values = independent variable values; range values = dependent variable values)

Reflection

Have students work in groups of two or three to model the problems below with an equation, table, and graph. Have them include 5 data points for each story.

1. Joe rents inline skates to customers on the Boardwalk in Virginia Beach. He charges a $5.00 basic rental fee and $1.50 per hour to use them.
2. When a certain city gives out speeding tickets, it charges a basic fine of $75.00 plus $15.00 for every mile the offender was going over the speed limit.

Name: ______________________

Warm-up

Match the verbal expression with its algebraic representation.

1.	$n + 3$	a.	one-third of a number
2.	$3n$	b.	3 added to a number
3.	$\frac{1}{3}n$	c.	a number multiplied by 3
4.	$3 - n$	d.	3 subtracted from a number
5.	$\frac{3}{n}$	e.	a number subtracted from 3
6.	$n - 3$	f.	3 divided by a number

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

Name: ANSWER KEY

Warm-up

Match the verbal expression with its algebraic representation.

1.	$n + 3$	a.	one-third of a number
2.	$3n$	b.	3 added to a number
3.	$\frac{1}{3}n$	c.	a number multiplied by 3
4.	$3 - n$	d.	3 subtracted from a number
5.	$\frac{3}{n}$	e.	a number subtracted from 3
6.	$n - 3$	f.	3 divided by a number

1. b
2. c
3. a
4. e
5. f
6. d

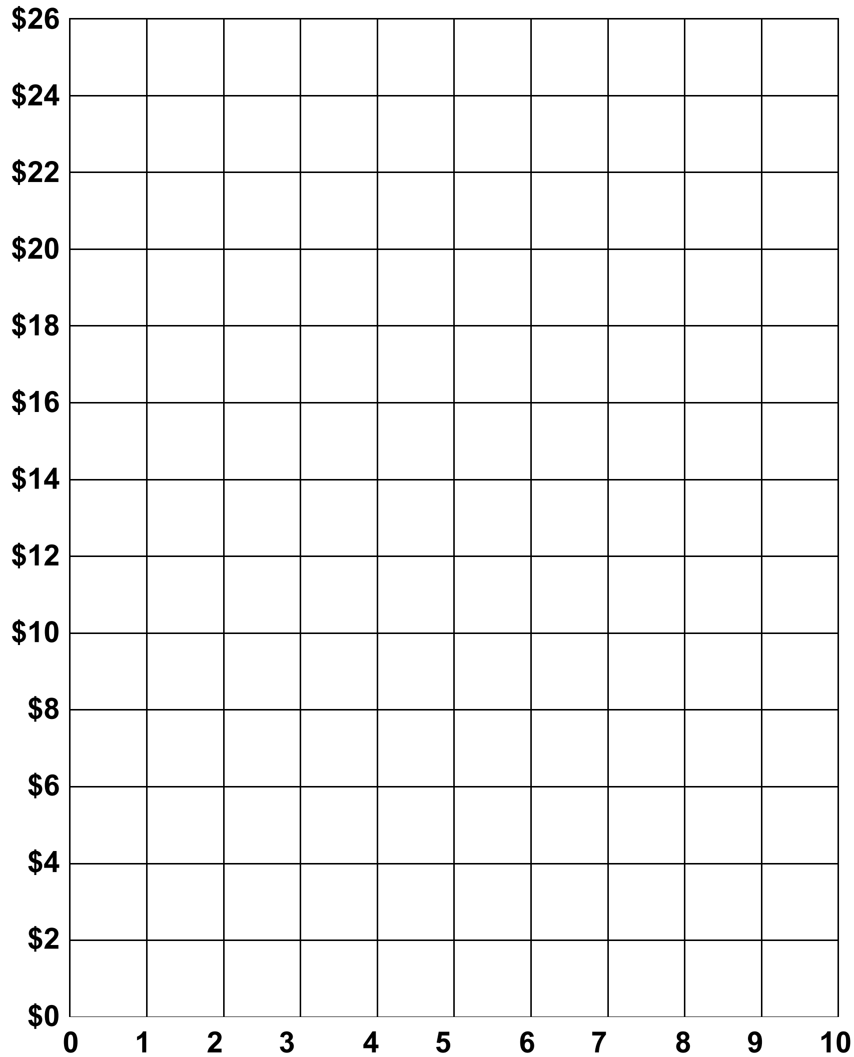
Name: ______________________

Using Equations in the Real World

Using the equation **0.5*n* + 15 = *m***, fill in the amount of money earned when delivering each number of newspapers.

Number of Newspapers Delivered (*n*)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Amount of Money Earned (*m*)										

Make a graph of the data you filled in above.



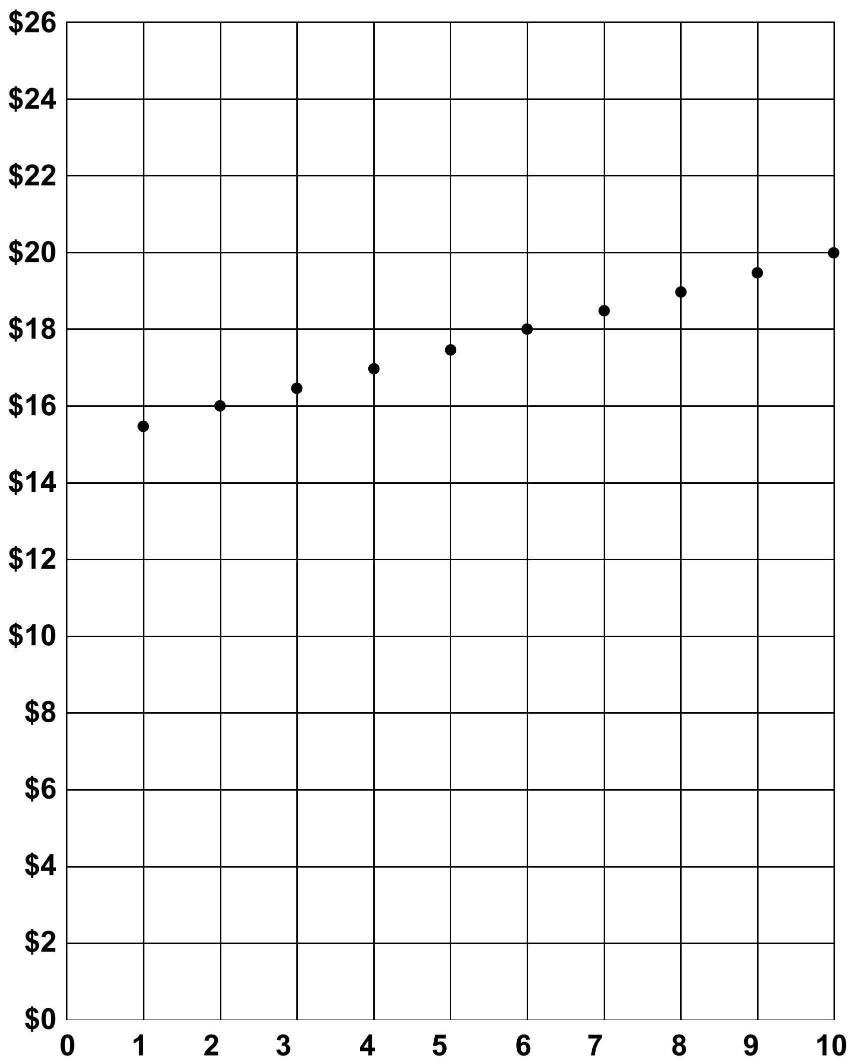
Name: <u>ANSWER KEY</u>

Using Equations in the Real World

Using the equation **0.5*n* + 15 = *m***, fill in the amount of money earned when delivering each number of newspapers.

Number of Newspapers Delivered (*n*)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Amount of Money Earned (*m*)	$15.50	$16.00	$16.50	$17.00	$17.50	$18.00	$18.50	$19.00	$19.50	$20.00

Make a graph of the data you filled in above.



✱ SOL 8.5

Prerequisite SOL

5.10, 6.10, 6.14

Lesson Summary

Using physical representations of square numbers, students investigate the relationship that exists between perfect squares and their square roots. (45 minutes)

Materials

Scissors
"Squares Template" handouts
Colored pencils
Calculators
"Perfect Squares" handouts
White boards (optional)
"Square Roots" worksheets

Vocabulary

perfect square. A whole number whose square root is a whole number.

Warm-up

Write the following incomplete sequences on the board. Ask students to complete the sequences and explain them.

1, 3, 6, 10, __, __, __, __	(Answer: 15, 21, 28, 36. The terms are increasing by 1 more than each previous increase.)
1, 1, 2, 3, 5, 8, __, __, __, __	(Answer: 13, 21, 34, 56. The terms are increasing by the sum of the previous two terms.)
1, 3, 5, 7, __, __, __, __	(Answer: 9, 11, 13, 15. The terms are increasing by 2.)
3/2, 2, 5/2, 3, 7/2, __, __, __, __	(Answer: 4, 9/2, 5, 11/2. The numerator of each fraction is increasing by 1, and fractions are being simplified.)
9, 23, 37, 51, 65, __, __, __, __	(Answer: 79, 93, 107, 121. The terms are increasing by 14.)

When students are finished, have them check their answers with a partner. Help partners resolve any difference between their answers to any sequence problem.

Lesson

1. Make enough copies of the "Squares Template" on card stock or heavy construction paper for each student to have two sheets.
2. Give each student a pair of scissors and two sheets of the "Squares Template." Have the students cut out the squares from one of the templates so that they each have 56 individual squares.
3. Ask students to construct as many different sizes of larger squares out of their individual squares as they can. For each one they make, they should copy and shade it on the intact template, using a different color for each square.
4. After recording their squares, ask students to write the area of each larger square below the square on the template. (They should get 4, 9, 25, 36, and 49.)
5. Conduct a class discussion. Begin by asking the class what areas their squares have, and record the areas on the board. Ask the students if these numbers look familiar.
6. Ask the students to take 15 individual squares and create a large square. Is this possible? (No) Have them try 7, 8, and 3. Once the students are convinced that these numbers of individual squares cannot produce larger squares, write "15, 7, 8, 3" on the board apart from the perfect square numbers. Ask the students what is special about 4, 9, 16, 25, and 36 that is not true of 15, 7, 8, and 3. The answers should be similar to: "4, 9, 16, 25, and 36 squares can make a larger square, but 15, 7,

8, and 3 squares cannot." Once the students come to that conclusion, name the special numbers *perfect squares*.

7. Once the perfect squares are named, point out the factors that are the square roots. Associate the square roots with the length of one side of each larger square. Ask students to identify in writing the perfect square numbers and the square roots for each larger square.
8. Have students work in pairs to answer the following questions: "The first six perfect square numbers are 1, 4, 9, 16, 25, and 36. What are the next five? How did you find your answers?" Assist partners in finding a strategy, if necessary.
9. Have the groups share their answers.
10. Ask students for the square roots of a number you call out. Begin with three or four perfect squares. Then ask for the square root of 15. Students should have difficulty. At this point, pass out the "Perfect Squares" list handouts, and ask the students to find where 15 would fall on the list. (The square root of 15 falls between 4 [the square root of 16] and 5 [the square root of 25].)
11. If time permits or in the next lesson, distribute white boards to students. Call out a number that is not a perfect square, and ask students to estimate the square root of that number by giving the two whole-number square roots it falls between. Allow students to consult their list of perfect squares. Have students write their answers on the white board and hold them up for you to check. Continue for as long as needed. This is a good activity to repeat daily as a brief review.
12. Conclude the lesson by having the students complete the "Square Roots" worksheet for review.

Reflection

Have students explain in writing how a perfect square and a square root relate to the area of a square. Allow them to use drawings in their explanations.

Squares Template

Perfect Squares

1

4

9

16

25

36

49

64

81

100

121

144

Name: ________________________

Square Roots

Approximate the following square roots. Between which two perfect square numbers does the answer lie?

1. $\sqrt{33}$ ________________
2. $\sqrt{46}$ ________________
3. $\sqrt{26}$ ________________
4. $\sqrt{62}$ ________________
5. $\sqrt{87}$ ________________

Use a calculator to determine whether each of the following numbers is a perfect square. Support your answer.

1. 96 ___________
2. 132 ___________
3. 529 ___________

Find each of the following measurements.

1. Find the length of the side of a square whose area is 144 in^2.

   ____

2. Find the area of a square whose side length is 10 cm.

   ____

Name: <u>ANSWER KEY</u>

Square Roots

Approximate the following square roots. Between which two perfect square numbers does the answer lie?

1. $\sqrt{33}$ __between 5 and 6__
2. $\sqrt{46}$ __between 6 and 7__
3. $\sqrt{26}$ __between 5 and 6__
4. $\sqrt{62}$ __between 7 and 8__
5. $\sqrt{87}$ __between 9 and 10__

Use a calculator to determine whether each of the following numbers is a perfect square. Support your answer.

1. 96 __no__
2. 132 __no__
3. 529 __yes (23)__

Find each of the following measurements.

1. Find the length of the side of a square whose area is 144 in^2.

   __12 in.__

2. Find the area of a square whose side length is 10 cm.

   __100 cm^2__

✱ SOL 8.14, 8.16, 8.17, 8.18

Prerequisite SOL

8.18

Lesson Summary

Students investigate and observe patterns in tables and graphs. Using construction-paper "bridges" and pennies as weights, they make predictions, run tests, and record data in tables and graphs, which they then analyze for patterns. (45 minutes)

Materials

Construction paper	Pennies	Two textbooks per group (any type)
Small paper cups	Graph paper	

Vocabulary

independent variable. The value that is manipulated or changed on purpose.
dependent variable. The value that responds to or is affected by the change.

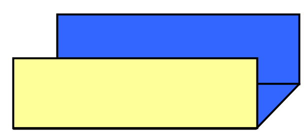
Warm-up

Write the following verbal expressions on the board, and ask students to translate them into algebraic expressions and write their expressions on paper. When everyone is finished, let students work with a partner to check their answers. Help partners resolve discrepancies as needed.

1. A number minus ten: ________ (n – 10)
2. Two more than a number: ________ (2 + n)
3. A number plus sixteen: ________ (n + 16)
4. A number divided by six: ________ (n ÷ 6)
5. Six times a number: ________ (6n)

Lesson

1. Lead a discussion about bridge building and the need for carefully designed bridges.
2. Divide the class into small groups of 2 or 3 students each, and pass out materials.
3. Explain to the students that they will simulate building a bridge and testing its strength. Each group will build a model bridge, using construction paper for the bridge and two textbooks as the bridge's supports. They will use pennies as weights to determine how much weight the bridge can support.
4. Instruct the groups to fold a piece of construction paper into three congruent sections with the folds parallel to the long side of the paper.



5. Have groups rest the ends of their folded paper "bridge" on two books, each of which is about 1.5 in. high. The bridge should overlap each book by 1 in.
6. Have each group place the cup in the center of the bridge and slowly add pennies until the bridge collapses. Have them write down the number of pennies it took to collapse the bridge.
7. After all groups have finished this experiment, ask the groups to predict what they think will happen if the bridge were made of two thicknesses of paper. What would happen if the bridge were three thicknesses?
8. Discuss methods the groups might use to record data when they try the experiment with different thicknesses of paper. If the students do not suggest a chart, suggest it yourself.
9. Have students create a chart like the one at right and record the number of pennies used for the first experiment.

# of sheets of paper	**1**	**2**	**3**	**4**
# of pennies				

10. Allow students to repeat the experiment with two sheets of paper, with three sheets of paper, and with four sheets of paper, and have them record their findings for each.

11. It is likely that each group's data will be slightly different due to the nature of the experiment. Ask students why the data might vary. Answers should include the way the paper was folded and whether the pennies were placed in the cup or dropped. Have students discuss how these variables might be controlled in order to obtain more consistent results.
12. When the students have completed their experiments, ask them, "Did the number of pennies needed depend on the number of sheets (thickness) of the paper, or did the paper thickness depend on the number of pennies?" (The number of pennies needed depended on the number of sheets of paper.) Identify the number of pennies as the dependent variable and the number of sheets of paper as the independent variable. Ask the students to write these terms in the appropriate locations on the table.
13. Pass out graph paper to each student. Remind them of the convention of letting the x-axis represent the independent variable and the y-axis represent the dependent variable. Have students graph their data. Walk around and provide assistance as needed.
14. Display the graphs so that all students can see them. Have students compare the results. While the graphs will not be identical, students should see strong similarities. Ask them why. (Because the more sheets of paper you use to construct the bridge, the more pennies it takes to make the bridge collapse.)

Reflection

Have students describe a real-world situation in which independent and dependent variables are important. Tell them to base the situation and the resulting experiment on having five types of bubble gum. Have them explain how they would set up their experiment and to identify the independent and dependent variables.

✱ SOL 8.14, 7.8, 8.7

Prerequisite SOL
5.21, 6.21, 6.23, 7.20, 7.12, 7.22

Lesson Summary
Students represent a function, using a table, an equation, and a graph. They use a physical model to demonstrate an understanding of these representations. They build models of cubes and record the growth in volume. (60 minutes)

Materials
An example of a rectangular prism (e.g., a box, carton)
Linking cubes
"Baby-Cube Problem Recording Sheet" handouts
Graph paper
Calculators

Vocabulary
polyhedron. A three-dimensional figure in which all the faces are polygons.
rectangular prism. A prism with six rectangular faces.
volume. The number of cubic units it takes to fill a three-dimensional figure.

Warm-up
Hold up an example of a rectangular prism. Examples could include a cereal box, a small carton, or any container with six rectangular faces. Ask the students the following questions:
- How many faces does a rectangular prism have? (6)
- How many vertices does a rectangular prism have? (8)
- How many edges does a rectangular prism have? (12)

Lesson
1. Hold up a single linking cube, and ask the students to name this figure. *Incorrect* responses might include "square," "rectangular pyramid," or "box." Remind students that a square is two-dimensional, and a box is a common name for a cube but is not necessarily geometric in nature. In addition to "cube," c*orrect* responses might include "polyhedron" (any three-dimensional figure in which all faces or surfaces are polygons) or "rectangular prism" (a polyhedron in which all six faces are rectangles).
2. Tell students that the single cube you are holding represents a "baby cube" at one year of age. Ask how you could construct a "baby cube" at two years of age. Students might not immediately understand the pattern and might suggest a two-cube rectangular prism, which is not a cube. If so, remind them that a cube must have all equal dimensions (length, height, and width), so a "baby cube" at two years of age will have all dimensions equal to two units. Have students begin recording the volume of the "aging" cubes on a table like the one below:

Age	Volume
1	1 cubic unit
2	8 cubic units
3	27 cubic units

3. Let the students continue building cubes until they discover that the volume of a cube is equal to the number of small cubes that make up the larger cube.
4. When all tables are complete, have the students look at their results and try to produce an equation for finding the volume at any age. If they have difficulty, have them say the process in words: "To get the volume, you multiply the length times the width times the height. Help them translate this into an algebraic expression with variables ($x \bullet x \bullet x$) and then to x^3. Point out that the length, width, and height dimensions for each age are the same as the age; therefore, if you know the age, you know the value of x, and you can find the volume.

5. Now that students have a table and an equation, distribute graph paper. Have students graph the independent variable on the *x*-axis and the dependent variables on the *y*-axis. Discuss why the volume is the dependent variable.
6. After the graphs are complete, have a discussion about how the equation, the table, and the graph are all representations of the volume of the cube at different ages.

Reflection

Have students imagine that a sick friend has stayed home, has missed the lesson, and needs to understand how an equation, a table, and a graph are really three different representations of the same concept. Ask, “How would you explain this to your friend over the telephone?” Have students write down their phone conversation.

Name: ________________________

Baby-Cube Problem Recording Sheet

Age	Volume
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
n	

Name: <u>ANSWER KEY</u>

Baby-Cube Problem Recording Sheet

Age	Volume
1	1 cubic units
2	8 cubic units
3	27 cubic units
4	64 cubic units
5	125 cubic units
6	216 cubic units
7	343 cubic units
8	512 cubic units
9	729 cubic units
10	1,000 cubic units
11	1,331 cubic units
12	1,728 cubic units
13	2,197 cubic units
14	2,744 cubic units
15	3, 375 cubic units
n	n^3 cubic units

✱ SOL 8.15

Prerequisite SOL
7.19, 7.20, 7.21, 7.22

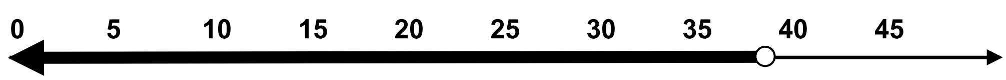
Lesson Summary
Students are presented with a real-world problem involving the side lengths of triangles. (60 minutes)

Materials
“Naming Triangles” worksheets
“Tricky Triangles Recording Sheet” handouts
Pipe cleaners
Rulers

Vocabulary
scalene triangle. A triangle in which no two sides are the same length.
isosceles triangle. A triangle in which two sides have the same length.
equilateral triangle. A triangle in which all three sides are of equal length.
acute triangle. A triangle with all three angles less than 90°.
right triangle. A triangle with one 90° angle.
obtuse triangle. A triangle with one angle greater than 90°.

Warm-up
Hand out the “Naming Triangles” worksheets, and have students complete them. Discuss the answers as a class.

Lesson
1. Pass out the “Tricky Triangles Recording Sheet” handouts, pipe cleaners, and rulers. Tell students that they should try to construct triangles with the side lengths listed in the table. As the students find some of the triangles impossible, have them conjecture why some are possible and some are not. Guide them to see the special relationship between any two sides of a triangle and the third side.
2. Following their hands-on experimentation, give the students the triangle inequality theorem, which states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.
3. Once students are clear on this relationship between any two sides of a triangle and the third side, present the following scenario to the students: “If a triangle has a side length of 24 cm, and another side measures 15 cm, what lengths can the other side be?” (Anything less than 39 cm)
4. Ask the students if there is a way to write the answer to this question, using a variable. Possible answers could include “The letter x must represent a number less than the sum of 24 and 15.” Once the students state the inequality in words, help them transfer this “math sentence” into an algebraic inequality: $x < 24 + 15$. Explain to students that this is referred to as an *inequality* and that the value of x has more than one correct solution.
5. Explain to students that solving an inequality is the same process as solving an equation. Have students solve this inequality by combining like terms (adding the two values): $x < 39$. Discuss what this means in relation to the triangle example. (The length of the third side must be *less than* 39 cm.)
6. Show the students a number line like the one shown below:



7. Demonstrate how to graph $x < 39$. The circle below the number line is open to denote that the inequality cannot equal the number. If the inequality had read $x \leq 39$, the circle would be shaded.

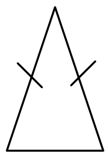
Reflection

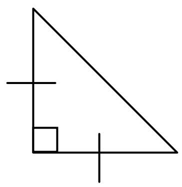
Have students explain in a written paragraph the difference between using an open circle and using a shaded circle in graphing an inequality. (An open circle is used to graph an inequality when the inequality stands alone, while a shaded circle is used to graph an inequality when the inequality reads “less than or equal to” or “greater than or equal to.”)

Name: ____________________________

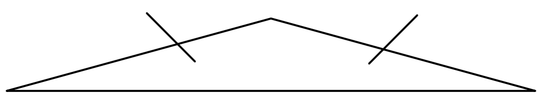
Naming Triangles

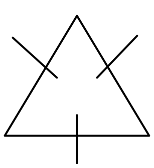
Name each triangle by its angles and sides.

1. 

2. 

3. 

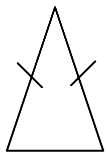
4. 

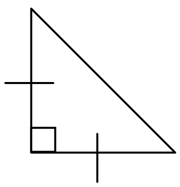
5. 

Name: ANSWER KEY

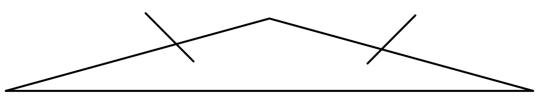
Naming Triangles

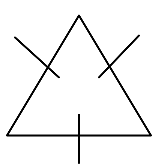
Name each triangle by its angles and sides.

1.  isosceles, acute

2.  isosceles, right

3.  scalene, right

4.  isosceles obtuse

5.  equilateral, acute

Name: ________________________

Tricky Triangles Recording Sheet

Using the pipe cleaners, try to construct triangles with the given side lengths. If a triangle with the given side lengths cannot be made, answer "no."

Side Lengths	Can a triangle be made? Yes or No
5 cm, 5 cm, 8 cm	
5 in., 8 in., 8 in.	
8 cm, 5 cm, 15 cm	
5 in., 6 in., 10 in.	
11 cm, 12 cm, 14 cm	
8 in., 8 in., 8 in.	

Based on your observations, what must be true in order to construct a triangle?

Name: ANSWER KEY

Tricky Triangles Recording Sheet

Using the pipe cleaners, try to construct triangles with the given side lengths. If a triangle with the given side lengths cannot be made, answer "no."

Side Lengths	Can a triangle be made? Yes or No
5 cm, 5 cm, 8 cm	yes
5 in., 8 in., 8 in.	yes
8 cm, 5 cm, 15 cm	no
5 in., 6 in., 10 in.	yes
11 cm, 12 cm, 14 cm	no
8 in., 8 in., 8 in.	yes

Based on your observations, what must be true in order to construct a triangle?

The triangle inequality theorem states that the sum of the lengths of any two sides of a triangle is at most greater than the length of the third side of that triangle.

✱ SOL 8.15

Prerequisite SOL

7.19, 7.20, 7.21, 7.22

Lesson Summary

Students model two-step equations, using Algeblocks™. (2 to 4 class periods)

Materials

Classroom set of Algeblocks™
A copy of the manual "Middle School Algeblocks™"

Warm-up

If students are not familiar with Algeblocks™, introduce them during the warm-up. A model of how to introduce the materials to students can be found in the streaming video at http://www.vdoe.whro.org/A_Blocks05/index.html. See the "Fundamentals" segment, which provides a model of the lesson on introducing the materials to the students. This will take one class period. (Note: All middle and high schools in Virginia received sets of Algeblocks™ and copies of the manual "Middle School Algeblocks™" in 2000–2001 from the Virginia Department of Education.)

Students will need the skills shown in the video segments on "Integers" and "Integer Operations" before they can use the mats to solve two-step equations. Assess your students' level of experience and deliver the prerequisite lessons, using the "Middle School Algeblocks™" manual and the lessons demonstrated in the video.

Lesson

1. Depending on your experience with Algeblocks™, it is suggested that you reference the streaming video http://www.vdoe.whro.org/A_Blocks05/index.html. The segments "Working with Integers," "Integer Operations," and "Adding and Subtracting Polynomials" will be most beneficial.
2. Work with students in completing lessons 6-1 through 6-8 from the manual "Middle School Algeblocks™."
3. For additional practice, select equations from the students' textbook, and ask them to solve the equations, using Algeblocks™. Keep in mind that the Algeblocks™ are a simply means to an end: if students can write the solution steps algebraically without relying on the Algeblocks™, allow them to do so.

Reflection

Ask students to draw the Algeblocks™ solution steps to the equation $3x - 1 = 2$.